

Physics data booklet

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Diploma Programme Physics data booklet

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Fundamental constants

| Quantity | Symbol | Approximate value |
|---------------------------------------------|--------------|------------------------------------------------------------------------------------|
| Acceleration of free fall (Earth's surface) | g | 9.81 m s^{-2} |
| Gravitational constant | G | $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ |
| Avogadro's constant | N_A | $6.02 \times 10^{23} \text{ mol}^{-1}$ |
| Gas constant | R | $8.31 \text{ J K}^{-1} \text{ mol}^{-1}$ |
| Boltzmann's constant | k_B | $1.38 \times 10^{-23} \text{ J K}^{-1}$ |
| Stefan-Boltzmann constant | σ | $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ |
| Coulomb constant | k | $8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ |
| Permittivity of free space | ϵ_0 | $8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ |
| Permeability of free space | μ_0 | $4\pi \times 10^{-7} \text{ T m A}^{-1}$ |
| Speed of light in vacuum | c | $3.00 \times 10^8 \text{ m s}^{-1}$ |
| Planck's constant | h | $6.63 \times 10^{-34} \text{ J s}$ |
| Elementary charge | e | $1.60 \times 10^{-19} \text{ C}$ |
| Electron rest mass | m_e | $9.110 \times 10^{-31} \text{ kg} = 0.000549 \text{ u} = 0.511 \text{ MeV c}^{-2}$ |
| Proton rest mass | m_p | $1.673 \times 10^{-27} \text{ kg} = 1.007276 \text{ u} = 938 \text{ MeV c}^{-2}$ |
| Neutron rest mass | m_n | $1.675 \times 10^{-27} \text{ kg} = 1.008665 \text{ u} = 940 \text{ MeV c}^{-2}$ |
| Unified atomic mass unit | u | $1.661 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV c}^{-2}$ |
| Solar constant | S | $1.36 \times 10^3 \text{ W m}^{-2}$ |
| Fermi radius | R_0 | $1.20 \times 10^{-15} \text{ m}$ |

Metric (SI) multipliers

| Prefix | Abbreviation | Value |
|--------|--------------|------------|
| peta | P | 10^{15} |
| tera | T | 10^{12} |
| giga | G | 10^9 |
| mega | M | 10^6 |
| kilo | k | 10^3 |
| hecto | h | 10^2 |
| deca | da | 10^1 |
| deci | d | 10^{-1} |
| centi | c | 10^{-2} |
| milli | m | 10^{-3} |
| micro | μ | 10^{-6} |
| nano | n | 10^{-9} |
| pico | p | 10^{-12} |
| femto | f | 10^{-15} |

Unit conversions

$$1 \text{ radian (rad)} \equiv \frac{180^\circ}{\pi}$$

$$\text{Temperature (K)} = \text{temperature (}^\circ\text{C)} + 273$$

$$1 \text{ light year (ly)} = 9.46 \times 10^{15} \text{ m}$$


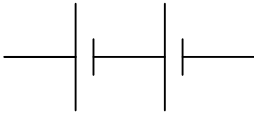

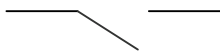
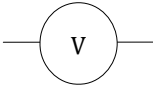
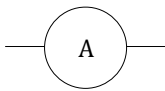
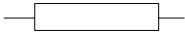
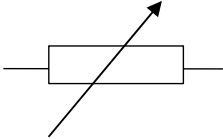
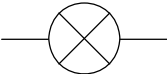
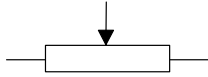
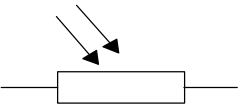
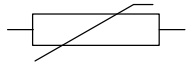

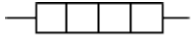
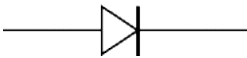
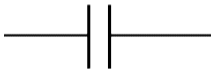
$$1 \text{ parsec (pc)} = 3.26 \text{ ly}$$

$$1 \text{ astronomical unit (AU)} = 1.50 \times 10^{11} \text{ m}$$

$$1 \text{ kilowatt-hour (kWh)} = 3.60 \times 10^6 \text{ J}$$

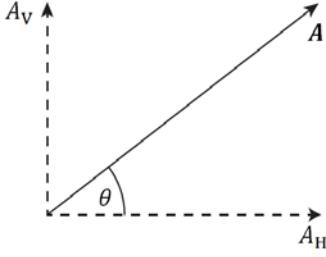
$$hc = 1.99 \times 10^{-25} \text{ J m} = 1.24 \times 10^{-6} \text{ eV m}$$

Electrical circuit symbols

| | | | |
|--------------------------------|-------------------------------------------------------------------------------------|-------------------|---------------------------------------------------------------------------------------|
| cell |  | battery |  |
| ac supply |  | switch |  |
| voltmeter |  | ammeter |  |
| resistor |  | variable resistor |  |
| lamp |  | potentiometer |  |
| light-dependent resistor (LDR) |  | thermistor |  |
| transformer |  | heating element |  |
| diode |  | capacitor |  |

Equations—Core

Note: All equations relate to the magnitude of the quantities only. Vector notation has not been used.

| Sub-topic 1.2 – Uncertainties and errors | Sub-topic 1.3 – Vectors and scalars |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>If: $y = a \pm b$ then: $\Delta y = \Delta a + \Delta b$</p> <p>If: $y = \frac{ab}{c}$ then: $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$</p> <p>If: $y = a^n$ then: $\frac{\Delta y}{y} = \left n \frac{\Delta a}{a} \right$</p> |  <p>$A_H = A \cos \theta$ $A_V = A \sin \theta$</p> |
| Sub-topic 2.1 – Motion | Sub-topic 2.2 – Forces |
| <p>$v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{(v + u)t}{2}$</p> | <p>$F = ma$ $F_f \leq \mu_s R$ $F_f = \mu_d R$</p> |
| Sub-topic 2.3 – Work, energy and power | Sub-topic 2.4 – Momentum and impulse |
| <p>$W = Fs \cos \theta$ $E_K = \frac{1}{2}mv^2$ $E_P = \frac{1}{2}k\Delta x^2$ $\Delta E_P = mg\Delta h$ power = Fv</p> <p>Efficiency = $\frac{\text{useful work out}}{\text{total work in}}$ = $\frac{\text{useful power out}}{\text{total power in}}$</p> | <p>$p = mv$ $F = \frac{\Delta p}{\Delta t}$ $E_K = \frac{p^2}{2m}$ Impulse = $F\Delta t = \Delta p$</p> |

| | |
|----------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Sub-topic 3.1 – Thermal concepts | Sub-topic 3.2 – Modelling a gas |
| $Q = mc\Delta T$ $Q = mL$ | $p = \frac{F}{A}$ $n = \frac{N}{N_A}$ $pV = nRT$ $\bar{E}_K = \frac{3}{2} k_B T = \frac{3}{2} \frac{R}{N_A} T$ |
| Sub-topic 4.1 – Oscillations | Sub-topic 4.4 – Wave behaviour |
| $T = \frac{1}{f}$ | $\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}$ $s = \frac{\lambda D}{d}$ |
| Sub-topic 4.2 – Travelling waves | Constructive interference: path difference = $n\lambda$ Destructive interference: path difference = $(n + \frac{1}{2})\lambda$ |
| $c = f\lambda$ | |
| Sub-topic 4.3 – Wave characteristics | |
| $I \propto A^2$ $I \propto x^{-2}$ $I = I_0 \cos^2 \theta$ | |
| Sub-topic 5.1 – Electric fields | Sub-topic 5.2 – Heating effect of electric currents |
| $I = \frac{\Delta q}{\Delta t}$ $F = k \frac{q_1 q_2}{r^2}$ $k = \frac{1}{4\pi\epsilon_0}$ $V = \frac{W}{q}$ $E = \frac{F}{q}$ $I = nAvq$ | Kirchoff's circuit laws: $\Sigma V = 0$ (loop) $\Sigma I = 0$ (junction) $R = \frac{V}{I}$ $P = VI = I^2 R = \frac{V^2}{R}$ $R_{\text{total}} = R_1 + R_2 + \dots$ $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$ $\rho = \frac{RA}{L}$ |
| Sub-topic 5.3 – Electric cells | Sub-topic 5.4 – Magnetic effects of electric currents |
| $\varepsilon = I(R + r)$ | $F = qvB \sin \theta$ $F = BIL \sin \theta$ |

| Sub-topic 6.1 – Circular motion | Sub-topic 6.2 – Newton's law of gravitation |
|----------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| $v = \omega r$ $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$ $F = \frac{mv^2}{r} = m\omega^2 r$ | $F = G \frac{Mm}{r^2}$ $g = \frac{F}{m}$ $g = G \frac{M}{r^2}$ |

| Sub-topic 7.1 – Discrete energy and radioactivity | Sub-topic 7.2 – Nuclear reactions |
|---------------------------------------------------|-----------------------------------|
| $E = hf$ $\lambda = \frac{hc}{E}$ | $\Delta E = \Delta m c^2$ |

Sub-topic 7.3 – The structure of matter

| Charge | Quarks | | | Baryon number |
|--------------------------------------------------------------------------------------------------------|--------|---|---|---------------|
| $\frac{2}{3}e$ | u | c | t | $\frac{1}{3}$ |
| $\frac{1}{3}e$ | d | s | b | $\frac{1}{3}$ |
| All quarks have a strangeness number of 0 except the strange quark that has a strangeness number of -1 | | | | |

| Charge | Leptons | | |
|----------------------------------------------------------------------------------|---------|-----------|------------|
| -1 | e | μ | τ |
| 0 | ν_e | ν_μ | ν_τ |
| All leptons have a lepton number of 1 and antileptons have a lepton number of -1 | | | |

| | Gravitational | Weak | Electromagnetic | Strong |
|------------------------|---------------|-----------------|-----------------|----------------|
| Particles experiencing | All | Quarks, leptons | Charged | Quarks, gluons |
| Particles mediating | Graviton | W^+, W^-, Z^0 | γ | Gluons |

| Sub-topic 8.1 – Energy sources | Sub-topic 8.2 – Thermal energy transfer |
|---------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Power = $\frac{\text{energy}}{\text{time}}$ Power = $\frac{1}{2} A \rho v^3$ | $P = e\sigma AT^4$ $\lambda_{\text{max}}(\text{metres}) = \frac{2.90 \times 10^{-3}}{T(\text{kelvin})}$ $I = \frac{\text{power}}{A}$ albedo = $\frac{\text{total scattered power}}{\text{total incident power}}$ |

Equations—AHL

| Sub-topic 9.1 – Simple harmonic motion | Sub-topic 9.2 – Single-slit diffraction | | | | | | | | |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------|----------------------|------------------------------------|------------------------------------|-------------------------------|----------------------------------|------------------------------|------------------------------|
| $\omega = \frac{2\pi}{T}$ $a = -\omega^2 x$ $x = x_0 \sin \omega t ; x = x_0 \cos \omega t$ $v = \omega x_0 \cos \omega t ; v = -\omega x_0 \sin \omega t$ $v = \pm \omega \sqrt{(x_0^2 - x^2)}$ $E_K = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$ $E_T = \frac{1}{2} m \omega^2 x_0^2$ Pendulum: $T = 2\pi \sqrt{\frac{l}{g}}$ Mass-spring: $T = 2\pi \sqrt{\frac{m}{k}}$ | $\theta = \frac{\lambda}{b}$ | | | | | | | | |
| | Sub-topic 9.3 – Interference | | | | | | | | |
| | $n\lambda = d \sin \theta$ Constructive interference: $2dn = (m + \frac{1}{2}) \lambda$ Destructive interference: $2dn = m\lambda$ | | | | | | | | |
| Sub-topic 9.4 – Resolution | Sub-topic 9.5 – Doppler effect | | | | | | | | |
| $\theta = 1.22 \frac{\lambda}{b}$ $R = \frac{\lambda}{\Delta\lambda} = mN$ | Moving source: $f' = f \left(\frac{v}{v \pm u_s} \right)$ Moving observer: $f' = f \left(\frac{v \pm u_o}{v} \right)$ $\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$ | | | | | | | | |
| Sub-topic 10.1 – Describing fields | Sub-topic 10.2 – Fields at work | | | | | | | | |
| $W = q\Delta V_e$ $W = m\Delta V_g$ | <table border="1"> <tbody> <tr> <td>$V_g = -\frac{GM}{r}$</td> <td>$V_e = \frac{kq}{r}$</td> </tr> <tr> <td>$g = -\frac{\Delta V_g}{\Delta r}$</td> <td>$E = -\frac{\Delta V_e}{\Delta r}$</td> </tr> <tr> <td>$E_P = mV_g = -\frac{GMm}{r}$</td> <td>$E_P = qV_e = \frac{kq_1q_2}{r}$</td> </tr> <tr> <td>$F_G = G \frac{m_1m_2}{r^2}$</td> <td>$F_E = k \frac{q_1q_2}{r^2}$</td> </tr> </tbody> </table> $v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$ $v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$ | $V_g = -\frac{GM}{r}$ | $V_e = \frac{kq}{r}$ | $g = -\frac{\Delta V_g}{\Delta r}$ | $E = -\frac{\Delta V_e}{\Delta r}$ | $E_P = mV_g = -\frac{GMm}{r}$ | $E_P = qV_e = \frac{kq_1q_2}{r}$ | $F_G = G \frac{m_1m_2}{r^2}$ | $F_E = k \frac{q_1q_2}{r^2}$ |
| $V_g = -\frac{GM}{r}$ | $V_e = \frac{kq}{r}$ | | | | | | | | |
| $g = -\frac{\Delta V_g}{\Delta r}$ | $E = -\frac{\Delta V_e}{\Delta r}$ | | | | | | | | |
| $E_P = mV_g = -\frac{GMm}{r}$ | $E_P = qV_e = \frac{kq_1q_2}{r}$ | | | | | | | | |
| $F_G = G \frac{m_1m_2}{r^2}$ | $F_E = k \frac{q_1q_2}{r^2}$ | | | | | | | | |

| Sub-topic 11.1 – Electromagnetic induction | Sub-topic 11.3 – Capacitance | |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------|
| $\Phi = BA \cos \theta$ $\varepsilon = -N \frac{\Delta \Phi}{\Delta t}$ $\varepsilon = Bvl$ $\varepsilon = BvlN$ | $C = \frac{q}{V}$ $C_{\text{parallel}} = C_1 + C_2 + \dots$ $\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$ | |
| Sub-topic 11.2 – Power generation and transmission | $C = \varepsilon \frac{A}{d}$ $E = \frac{1}{2} CV^2$ $\tau = RC$ $q = q_0 e^{-\frac{t}{\tau}}$ $I = I_0 e^{-\frac{t}{\tau}}$ $V = V_0 e^{-\frac{t}{\tau}}$ | |
| $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$ $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ $R = \frac{V_0}{I_0} = \frac{V_{\text{rms}}}{I_{\text{rms}}}$ $P_{\text{max}} = I_0 V_0$ $\bar{P} = \frac{1}{2} I_0 V_0$ $\frac{\varepsilon_p}{\varepsilon_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}$ | <th data-bbox="786 1059 1193 1104">Sub-topic 12.2 – Nuclear physics</th> | Sub-topic 12.2 – Nuclear physics |
| Sub-topic 12.1 – The interaction of matter with radiation | $E = hf$ $E_{\text{max}} = hf - \Phi$ $E = -\frac{13.6}{n^2} eV$ $mvr = \frac{nh}{2\pi}$ $P(r) = \psi ^2 \Delta V$ $\Delta x \Delta p \geq \frac{h}{4\pi}$ $\Delta E \Delta t \geq \frac{h}{4\pi}$ $R = R_0 A^{1/3}$ $N = N_0 e^{-\lambda t}$ $A = \lambda N_0 e^{-\lambda t}$ $\sin \theta \approx \frac{\lambda}{D}$ | |

Equations—Options

| Sub-topic A.1 – The beginnings of relativity | Sub-topic A.2 – Lorentz transformations |
|--------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $x' = x - vt$ $u' = u - v$ | $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ |
| Sub-topic A.3 – Spacetime diagrams | $x' = \gamma(x - vt) ; \Delta x' = \gamma(\Delta x - v\Delta t)$ $t' = \gamma\left(t - \frac{vx}{c^2}\right) ; \Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$ $u' = \frac{u - v}{1 - \frac{uv}{c^2}}$ $\Delta t = \gamma\Delta t_0$ $L = \frac{L_0}{\gamma}$ $(ct')^2 - (x')^2 = (ct)^2 - (x)^2$ |
| $\theta = \tan^{-1}\left(\frac{v}{c}\right)$ | |
| Sub-topic A.4 – Relativistic mechanics (HL only) | Sub-topic A.5 – General relativity (HL only) |
| $E = \gamma m_0 c^2$ $E_0 = m_0 c^2$ $E_K = (\gamma - 1)m_0 c^2$ $p = \gamma m_0 v$ $E^2 = p^2 c^2 + m_0^2 c^4$ $qV = \Delta E_K$ | $\frac{\Delta f}{f} = \frac{g\Delta h}{c^2}$ $R_s = \frac{2GM}{c^2}$ $\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$ |

| Sub-topic B.1 – Rigid bodies and rotational dynamics | Sub-topic B.2 – Thermodynamics |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\Gamma = Fr \sin \theta$ $I = \sum mr^2$ $\Gamma = I\alpha$ $\omega = 2\pi f$ $\omega_f = \omega_i + \alpha t$ $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\theta = \omega_i t + \frac{1}{2}\alpha t^2$ $L = I\omega$ $E_{Krot} = \frac{1}{2}I\omega^2$ | $Q = \Delta U + W$ $U = \frac{3}{2}nRT$ $\Delta S = \frac{\Delta Q}{T}$ $pV^{\frac{5}{3}} = \text{constant (for monatomic gases)}$ $W = p\Delta V$ $\eta = \frac{\text{useful work done}}{\text{energy input}}$ $\eta_{Carnot} = 1 - \frac{T_{cold}}{T_{hot}}$ |
| Sub-topic B.3 – Fluids and fluid dynamics (HL only) | Sub-topic B.4 – Forced vibrations and resonance (HL only) |
| $B = \rho_f V_f g$ $P = P_0 + \rho_f g d$ $Av = \text{constant}$ $\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant}$ $F_D = 6\pi\eta r v$ $R = \frac{vr\rho}{\eta}$ | $Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$ $Q = 2\pi \times \text{resonant frequency} \times \frac{\text{energy stored}}{\text{power loss}}$ |
| Sub-topic C.1 – Introduction to imaging | Sub-topic C.2 – Imaging instrumentation |
| $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ $P = \frac{1}{f}$ $m = \frac{h_i}{h_o} = -\frac{v}{u}$ $M = \frac{\theta_i}{\theta_o}$ $M_{\text{near point}} = \frac{D}{f} + 1; M_{\text{infinity}} = \frac{D}{f}$ | $M = \frac{f_o}{f_e}$ |
| | Sub-topic C.3 – Fibre optics |
| | $n = \frac{1}{\sin c}$ $\text{attenuation} = 10 \log \frac{I}{I_0}$ |
| | Sub-topic C.4 – Medical imaging (HL only) |
| | $L_1 = 10 \log \frac{I_1}{I_0}$ $I = I_0 e^{-\mu x}$ $\mu x_{\frac{1}{2}} = \ln 2$ $Z = \rho c$ |

| Sub-topic D.1 – Stellar quantities | Sub-topic D.2 – Stellar characteristics and stellar evolution |
|-------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------|
| $d \text{ (parsec)} = \frac{1}{p \text{ (arc-second)}}$ $L = \sigma AT^4$ $b = \frac{L}{4\pi d^2}$ | $\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$ $L \propto M^{3.5}$ |
| Sub-topic D.3 – Cosmology | Sub-topic D.5 – Further cosmology (HL only) |
| $z = \frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c}$ $z = \frac{R}{R_0} - 1$ $v = H_0 d$ $T \approx \frac{1}{H_0}$ | $v = \sqrt{\frac{4\pi G \rho}{3}} r$ $\rho_c = \frac{3H^2}{8\pi G}$ |