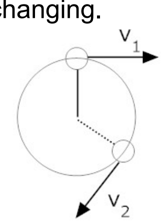
Circular Motion

Goin' around and around and ...

Uniform Circular Motion

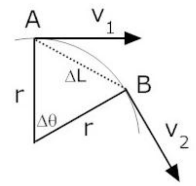
- An object that moves in a circle at constant speed v, is said to experience uniform circular motion.

- The magnitude of the linear velocity remains constant, but the direction is constantly changing.

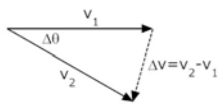


- Since acceleration is defined as the rate of change of velocity, a change in direction means that there is acceleration.

Deriving an Expression for Centripetal Acceleration



$$a = \frac{\Delta v}{\Delta t}$$



- During Δt the object moves from A to B
- The object covers the distance ΔL
- $\Delta v = v_2 - v_1$

- If we have a very small $\Delta\theta$ then…

$$\frac{\Delta v}{v} \approx \frac{\Delta L}{r}$$

- Similar triangles
- We assume that the magnitude of $v_1 = v_2$ (completely true if $\Delta t = 0$)

Solve for Δv

$$\Delta v = \frac{v \Delta L}{r}$$

Divide both sides by Δt

$$\frac{\Delta v}{\Delta t} = \frac{v \Delta L}{r \Delta t}$$

$\frac{\Delta v}{\Delta t}$ is acceleration

$\frac{\Delta L}{\Delta t}$ is speed

Therefore…

$$a = \frac{v^2}{r}$$

Centripetal (radial) **Acceleration**

(points towards the center)

- Circular motion is often described in terms of period (or frequency)
- Period (T)
  - The time of 1 revolution
- Frequency (f)
  - Number of revolutions per second
- Period and frequency are related:

$$T = \frac{1}{f}$$

One revolution around a circle is *2πr*

So…

$$v = \frac{2\pi r}{T}$$

Therefore…

$$a = \frac{4\pi^2 r}{T^2}$$

Example

- A 150 g ball at the end of a string is revolving in a horizontal circle of radius 0.60 m. The ball makes 2 revolutions in one second. What is its centripetal acceleration?

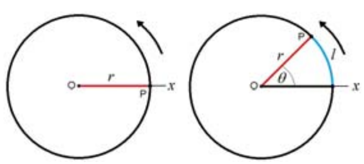
a = 95 ms^{-2}

Angular Quantities

- When an object moves in a circular path we can describe its position, velocity, and acceleration in terms of angle.

Angular Position

- To describe the angular position of an object, or how far it has rotated, we specify the angle θ of some particular line in the object with respect to a reference line in the object (x-axis)

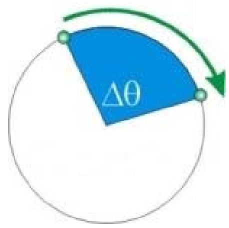


- The angle is measured in radians and is given by

$$\theta = \frac{l}{r}$$

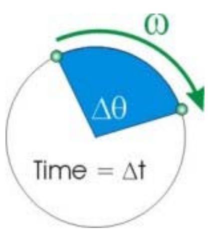
Angular Displacement

- The angle in radians through which a point has been rotated about a specified axis



Angular Velocity

- Change in angle per unit time

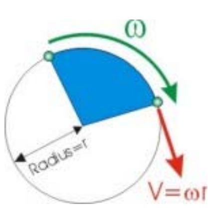


$$\omega = \frac{\Delta\theta}{\Delta t}$$

Units: rad s^{-1}

Linear Velocity

- Angular velocity times the radius of the circular path.



$$v = \omega r$$

Centripetal Force

- According to Newton's second law, an accelerating object must have a net force acting on it.
- For circular motion, this net (or total) force is called the **centripetal force**.

$$F = ma_c \text{ or } \boxed{F = \frac{mv^2}{r}} \text{ or } \boxed{F = m\omega^2 r}$$

Centripetal force always points towards the center of the circular path.