

Rotational Dynamics

Angular Quantities

- Until now we have been mainly concerned with translational motion
- When we have a rigid object rotating about a pivot point we can define angular quantities that are analogous with translational quantities:
 - Angular acceleration
 - Moment of inertia, torque, and angular momentum
 - Rotational kinetic energy, work, and power

Uniform Circular Motion

- When an object moves in a circle with constant linear speed (or angular speed) the net force acting on the object causes a centripetal force.

$$F = \frac{mv^2}{r}$$

or $F = m\omega^2 r$

ω is the angular velocity
(or frequency)

$$\omega = 2\pi f$$

Angular Acceleration

- Angular acceleration is defined as the rate of change of angular speed

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Units: rad s⁻²

- We derived equations to describe motion with constant acceleration

$$v = u + at \quad s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

- Similarly we have equations to describe motion with constant angular acceleration

$$\omega_f = \omega_i + \alpha t \quad \theta = \omega_i t + \frac{1}{2}\alpha t^2$$
$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

Example

- A wheel is rotated from rest with an angular acceleration of 8.0 rad s⁻². It accelerates for 5.0 s. Calculate the angular speed and the number of revolutions that the wheel has rotated through.
 - $\omega = 40$. rad s⁻¹
 - 15.9 revolutions

Moment of Inertia

- The moment of inertia, I , of a body is the rotational equivalent of the role played by mass in linear dynamics
- The moment of inertia of an object depends on the axis about which is rotated
- For a single point mass m rotating at a distance r about an axis, the moment of inertia is given by

$$I = mr^2$$

- For an object consisting of more than one point mass, the moment of inertia about a given axis can be calculated by adding the moments of inertia for each point mass

$$I = \sum mr^2$$

Examples

- Pendulum: $I_{\text{pendulum}} = ml^2$
- Solid sphere: $I_{\text{solid sphere}} = \frac{2}{5}mr^2$

Torque

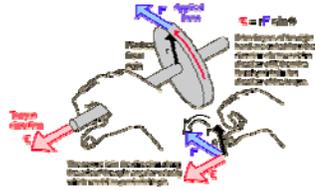
- Torque is a measure of how much a force acting on an object causes that object to rotate
- The object rotates about an axis or pivot point
- The distance from the pivot point to the point where the force acts is called the moment arm, and is denoted by r
- Only the component of the force that is perpendicular to the moment arm causes torque

$$\Gamma = Fr \sin \theta$$

Units: Nm

Direction of Torque

- The direction of the torque is given by a right hand rule



Couples

- A couple consists of a pair of equal and opposite forces that do not act in the same straight line

$$\Gamma = Fd$$

Where d is the perpendicular distance between the two forces

Newton's Laws for Angular Motion

- First Law
 - An object continues to remain stationary or to move at a constant angular velocity unless an external torque acts on it
- Second Law
 - $\Gamma = I\alpha$
- Third Law
 - If a body A applies a torque to body B, then body B applies an equal and opposite torque to body A

Rotational Equilibrium

- When an object is in rotational equilibrium total clockwise torque is equal to total anticlockwise torque
- This is often called the “principle of moments”

Angular Momentum

- The product of a body's moment of inertia and its angular velocity

$$L = I\omega$$

Units: $\text{kg m}^2 \text{ rad s}^{-1}$

Conservation of Angular Momentum

- The total angular momentum of a system remains constant providing no external torque acts on it

Rotational Kinetic Energy, Work and Power

- Kinetic Energy

$$E_{K,rot} = \frac{1}{2} I \omega^2$$

- Work $W = \Gamma \theta$

- Power $P = \Gamma \omega$

Rolling and Slipping

- If an object makes a perfectly frictionless contact with a surface it is impossible for the object to roll – it simply slides
- When there is friction the object can roll
- Since the point of contact between the rolling body and the surface on which it rolls is instantaneously stationary, the coefficient of static friction should be used in calculations involving rolling
- The point of contact must be stationary because it does not slide
