Kinematics

Vectors and Scalars

- Scalar
  - quantities that only contain magnitude and units.
  - Ex: time, mass, length, temperature
- Vector
  - quantities that contain magnitude and direction (as well as units).
  - Ex: displacement, velocity, force, acceleration

Vector Notation

- We represent a vector quantity by drawing an arrow above the letter.

$$\vec{d}$$

Direction

- The direction of the vector can be described in a number of ways:
  - Common terms (left/right, up/down, forward/backward)
  - Compass directions (north, south, east, west)
  - Number line, using positive and negative signs (+/-)
  - Coordinate system using angles of rotation from the horizontal axis

Adding Vectors

- There are two ways that we will use to add vectors:
  - Scale drawings
  - Algebraically

Scale Drawings

- We draw vectors as lines with an arrow head representing the tip of the vector

- The other end of the vector line is called the tail

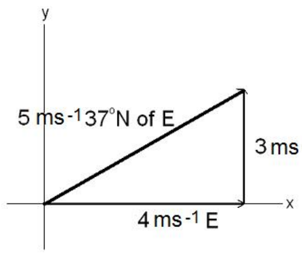
- Choose an appropriate scale
- Draw a line representing the first vector
- Draw a line representing the second vector starting from the tip of the first vector
- Continue until all vectors are drawn
- Join the tail of the first vector to the tip of the last vector
- Measure the length and angle of the joining line

- Example:
  - Add the following vectors: 5 ms^{-1} North and 10 ms^{-1} East

- We could also add these vectors using the Pythagorean theorem, sine laws, and cosine law

Components of Vectors

- Consider the following situation:
  - (4 ms^{-1} E) + (3 ms^{-1} N)



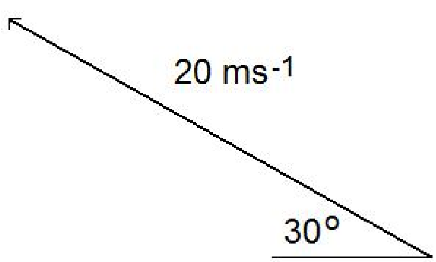
- Notice that
  - The vectors form a right angle triangle
  - The sides are related through trigonometry

$$5\sin 37 = 3$$
$$5\cos 37 = 4$$

- This is true for any vector

Example

- 20 ms^{-1} 30° N of W

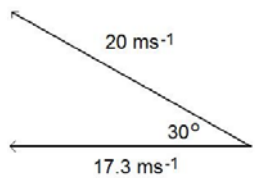


- We can find the horizontal side (called the horizontal component or x component) of the triangle by using cos

$$\cos 30^\circ = \frac{x}{20\ ms^{-1}}$$

$$x = (20\ ms^{-1})\cos 30^\circ$$

$$x = 17.3\ ms^{-1}$$

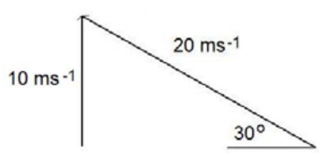


- We can find the vertical side (called the vertical component or y component) of the triangle by using sin

$$\sin 30^\circ = \frac{y}{20\ ms^{-1}}$$

$$y = (20\ ms^{-1})\sin 30^\circ$$

$$y = 10\ ms^{-1}$$



Adding with Components

- We can add vectors by using their components
  - Find the components of each vector
  - Add the x components together (remembering direction)
  - Add the y components together (remembering the direction)
  - Put the final vector together (using the Pythagorean therom)

Example

- Add the following vectors:
  - 30 ms^{-1} 25^o N of W
  - 50 ms^{-1} 40^o S of E

Definitions

- Displacement
  - the change in position of an object. How far the object is away from its starting position. Displacement is a vector quantity.
  - Symbol: s

- Velocity
  - signifies both speed and direction. It is a vector quantity.
  - The change in displacement with respect to time
  - Symbol: v

$$v = \frac{\Delta s}{\Delta t}$$

- Acceleration
  - how rapidly velocity changes. It is a vector quantity.
  - Change in velocity with respect to time
  - It is important to note that acceleration can occur if either speed **or** direction changes.
  - Symbol: a

$$a = \frac{\Delta v}{\Delta t}$$

What is the difference between average and instantaneous?

- Average
  - Measured over a period of time
- Instantaneous
  - Measured over a single infinitesimally small point in time
  - At one exact point in time
  - For example, a speedometer measures instantaneous velocity of a vehicle

Frames of Reference

- Any measurement of position, distance, or speed must be made with respect to a frame of reference.

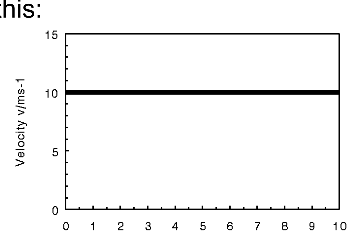
Example

- You are in a car traveling 80 kmh^{-1}. You notice a fly flying towards the front of the car at a speed of 5 kmh^{-1}.
- You are looking at the fly's speed from the reference frame of the car.
- To someone standing on the sidewalk the fly is traveling at a speed of 80 kmh^{-1} + 5 kmh^{-1} = 85 kmh^{-1} with respect to the ground.

- This is why it is always important to know the frame of reference.
- In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame should be specified whenever there might be confusion.
- The term **relative** is used in these cases.

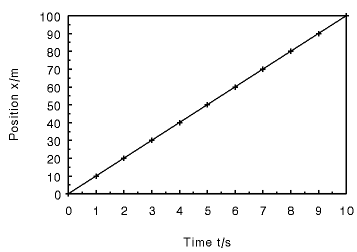
Graphical Representation of Motion

- Consider a car traveling at a constant velocity of 10 ms^{-1}. If we were to draw a graph of velocity versus time, it would look like this:



- It is also useful to graph position versus time.
- We will make the decision that when t=0, our position, x, will be 0.
- Since the car is moving with constant velocity, we can easily calculate how far the car will have traveled in 1s, 2s, 3s, etc.

- Plotting this gives us the following graph:

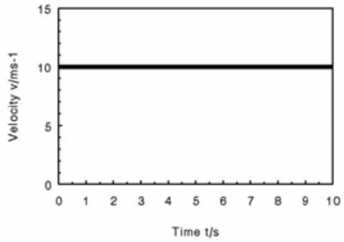


- Let's calculate the slope of this line:

$$slope = \frac{rise}{run} = \frac{90m - 10m}{9s - 1s} = \frac{80m}{8s} = 10ms^{-1}$$

- Notice that this is the same as the velocity.
- Therefore, the slope of a position (displacement) versus time graph is velocity.

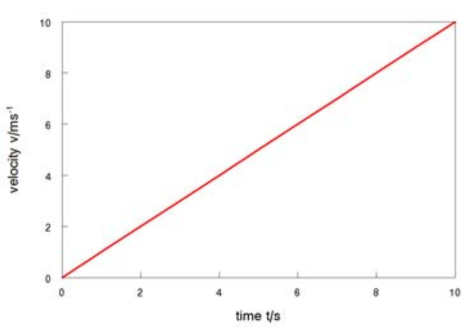
- Let's go back to our original velocity-time graph.



- How far does the car travel in 10 s?
  - 100 m

- Notice that the distance is the same as the area under the curve.
- In other words, the area under a velocity-time curve is displacement.

- Now let's consider a velocity-time graph of a car that is accelerating

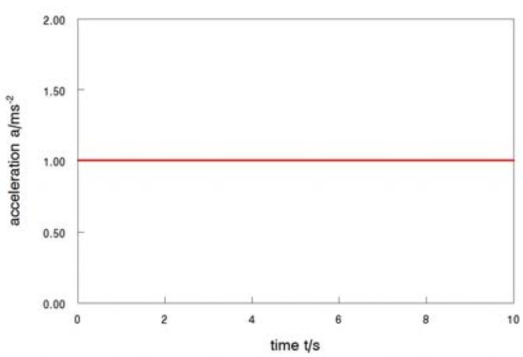


- Notice that the slope of the graph is equivalent to our definition of acceleration.

$$slope = \frac{rise}{run} = \frac{(10-0)\,\mathrm{ms^{-1}}}{(10-0)\,\mathrm{s}} = 1\,\mathrm{ms^{-2}}$$

$$a = \frac{\Delta \vec{v}}{\Delta t} = \frac{(10-0)\,\mathrm{ms^{-1}}}{(10-0)\,\mathrm{s}} = 1\,\mathrm{ms^{-2}}$$

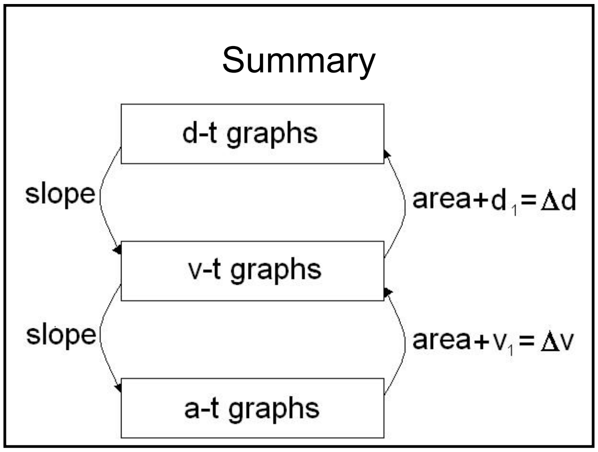
- A graph of this acceleration looks like this:



- Notice how the area under the curve is the same as the velocity.

$$area = (1\,\mathrm{ms^{-2}})(10\,\mathrm{s}) = 10\,\mathrm{ms^{-1}}$$

$$\Delta v = at = (1\,\mathrm{ms^{-2}})(10\,\mathrm{s}) = 10\,\mathrm{ms^{-1}}$$


Summary
d-t graphs
slope
area+d1=Δd
v-t graphs
slope
area+v1=Δv
a-t graphs