Simple Harmonic Motion

Oscillations

- Repetitive (periodic) motion
- Object moves back and forth around an equilibrium position

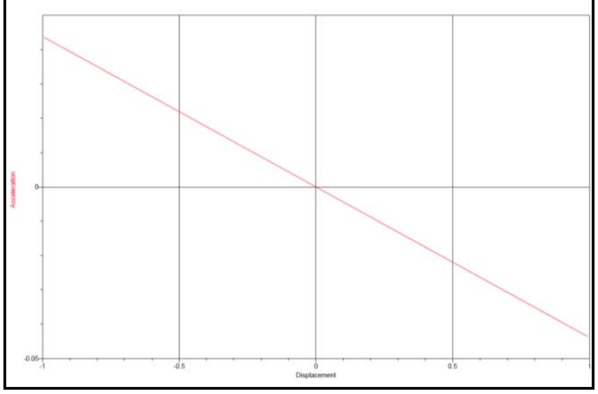
Simple Harmonic Motion

- Periodic motion that is sinusoidal in nature

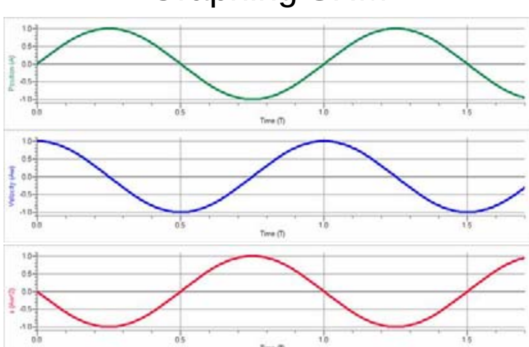
Conditions Required for SHM

- The acceleration causing the motion is proportional to the displacement of the object from its equilibrium position
- The accelerating force must be trying to restore the object to its equilibrium position

$$a \propto -x$$



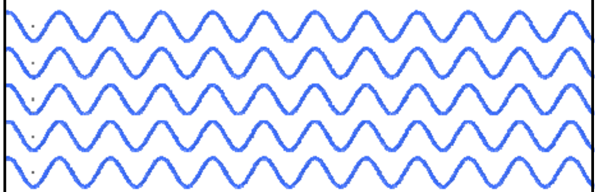
Graphing SHM



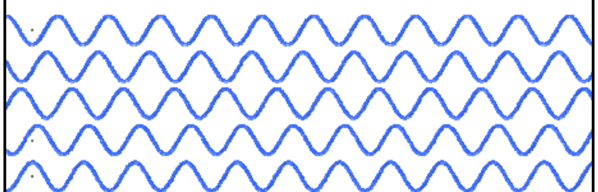
farraguttn.com/science/milligan/apphys/shmover.htm

Phase Difference

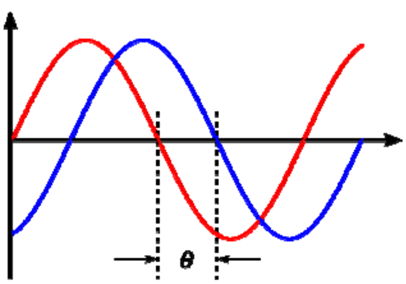
- When waves have the same frequency and starting point they are said to be in phase



- When waves of the same frequency have different starting points they are said to be out of phase

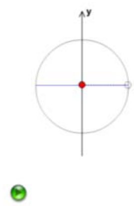


- The difference between starting points between waves that are out of phase is the phase difference or phase shift (measured in degrees or radians)

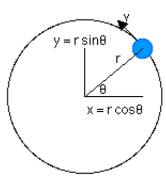


SHM and Circular Motion

- SHM is related to circular motion



- An object moving in a circle with constant angular velocity has an angular displacement given by $\Theta = \omega t$
- The projection of the x component of the motion at a given time is $y = r \sin\Theta$ or equivalently $y = r \sin \omega t$



Displacement, Velocity, & Acceleration

- Displacement can be represented by the equation:

$$x = x_0 \sin \omega t$$

- An equation for the velocity can be found by taking the derivative (calculus) of the displacement giving:

$$v = \omega x_0 \cos \omega t$$

- The acceleration will be the derivative of the velocity:

$$a = -\omega^2 x_0 \sin \omega t$$

- But $x=x_0 \sin \omega t$
- So…

$$\boxed{a = -\omega^2 x}$$

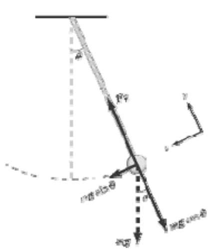
The Velocity Equation

- We can derive an equation for the velocity of the particle at a given point

$$\boxed{v = \pm\omega\sqrt{(x_0^2 - x^2)}}$$

Simple Pendulum

- The restoring force is perpendicular to the pendulum bob and string



$$-mg\sin\theta = ma$$

$$\sin\theta = \frac{x}{l}$$

x – displacement in x direction
l – length of pendulum

$$a = -\frac{gx}{l}$$

- This looks like the defining equation of SHM

$$a = -\left(\frac{g}{l}\right)x \qquad a = -\omega^2 x$$

- Therefore… $\omega^2 = \frac{g}{l}$
- Since the period for SHM is given by $T = \frac{2\pi}{\omega}$

the period of the pendulum is

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Mass and Spring

- The restoring force for a mass on a spring is $F=-kx$
- So… $-kx = ma \qquad a = -\left(\frac{k}{m}\right)x$
- Comparing this to the defining equation of SHM gives $\omega^2 = \frac{k}{m}$
- And therefore a period of $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

Energy

- A moving object has kinetic energy $E_k = \frac{1}{2}mv^2$
- The velocity of a particle in SHM at a given point is $v = \pm\omega\sqrt{(x_0^2 - x^2)}$
- Combining these equations gives us the kinetic energy at displacement x:

$$\boxed{E_k = \tfrac{1}{2}m\omega^2(x_0^2 - x^2)}$$

- The maximum kinetic energy occurs when the displacement is zero

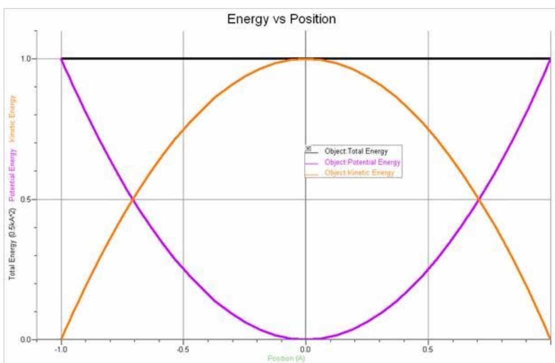
$$E_{k\,\max} = \tfrac{1}{2} m\omega^2 x_0^2$$

- This must be the total energy (as the potential energy at this point is zero) so we can say

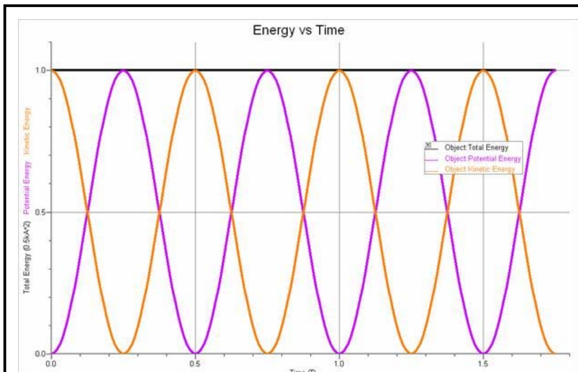
$$\boxed{E_T = \tfrac{1}{2} m\omega^2 x_0^2}$$

- So we can calculate the potential energy by subtracting kinetic energy from the total energy

$$E_P = E_T - E_k = \tfrac{1}{2} m\omega^2 x_0^2 - \tfrac{1}{2} m\omega^2 (x_0^2 - x^2) = \tfrac{1}{2} m\omega^2 x^2$$



farraguttn.com/science/milligan/apphys/shmover.htm



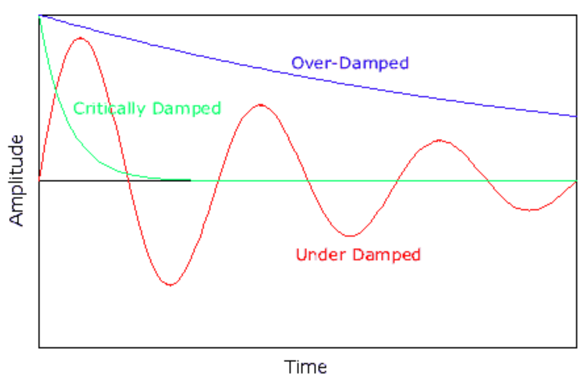
farraguttn.com/science/milligan/apphys/shmover.htm

Forced Vibrations

- All mechanical systems will vibrate when they are set in motion
- If a system is allowed to vibrate without any external forces being applied, it will vibrate at is natural frequency, f_0
- When resistive forces are present then the vibrations decay
- This is referred to as damping

Damping

- Underdamping
  - the amplitude gradually decreases until the oscillations stop
- Critical damping
  - The system returns to equilibrium in the shortest time possible with no oscillations
- Overdamping
  - The system returns to equilibrium with no oscillations but much slower than a critically damped system



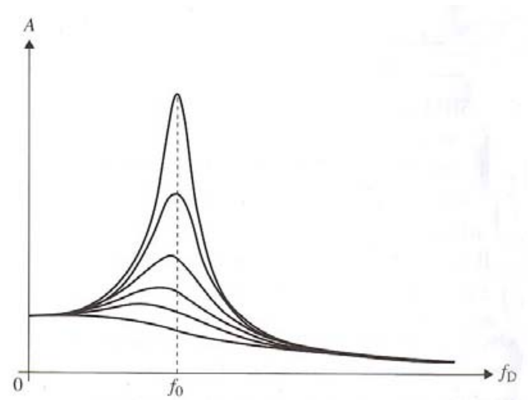
www.splung.com/content/sid/2/page/damped_oscillations

Forced Vibrations

- When an external force acts on a mechanical system, the force may have its own frequency of vibration, which may affect the motion of the mechanical system
- Forced vibrations are those that occur when a regularly changing external force is applied to a system resulting in the system vibrating at the same frequency as the force

Resonance

- When a mechanical system is forced to oscillate by a driving force that has the same frequency as the natural frequency of the mechanical system, it will vibrate with maximum amplitude.
- This is called resonance.
- The degree of damping will alter the amplitude response of the system.



Q factor

- The Q or “quality” factor is a criterion by which the sharpness of resonance can be assessed.

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$$

$$Q = 2\pi \times \text{resonant frequency} \times \frac{\text{energy stored}}{\text{power loss}}$$

- The Q factor is a numerical value with no units
- A system with a high Q factor is lightly damped
  - Energy dissipated per cycle is small
- The larger the Q factor, the sharper the resonance peak
- The Q factor is approximately the number of oscillations the system will make before its amplitude will decay to zero

- Some typical Q factors

Oscillator	Q factor
critically damped door	0.5
mass on spring	50
simple pendulum	200
oscillating quartz crystal	30 000

Resonance can be bad…

www.youtube.com/watch?v=iTFZNrTYp3k&safe=active

Resonance can also be good…

- Microwave ovens
  - Microwaves force oscillation of water molecules generating heat inside the food
- Radios
  - The tuner uses resonance to select the station
- Quartz oscillators
  - The quartz is forced with an electric current causing it to oscillate at a very specific frequency