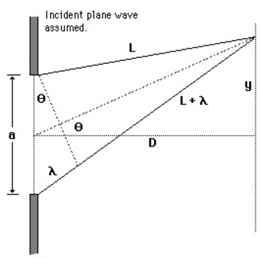
Wave Phenomena

Single Slit Diffraction

- Consider light incident on a slit of width **a**, in front of a screen **D** m away
- The length from the top of the slit to the screen is **L**
- The length from the bottom of the slit to the screen is **L +** λ



- From the diagram $\sin\theta = \frac{\lambda}{a}$
- For small angles, this is equivalent to $\boxed{\theta = \frac{\lambda}{a}}$
- This will calculate the angle of the first minimum
  - Note: The path length difference in the center of the slit with be $\lambda/2$. This will interfere with the waves from the edges resulting in total destructive interference

Example

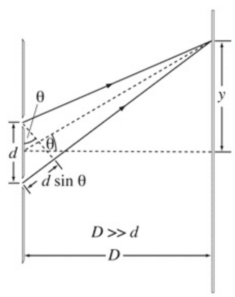
- Light of wavelength 550 nm is incident on a slit of width 1.5 μm. Calculate the width of the central maximum.

$$\theta = \frac{\lambda}{b} = \frac{550\times10^{-9}\,\text{m}}{1.5\times10^{-6}\,\text{m}} = 0.37 \text{ radians}$$

Central maximum = 0.74 radians

Double Slit Interference

- The path difference between the two slits is $d\sin\theta$
- For the point y to be a bright fringe (constructive interference) the path difference must be λ



- Therefore $d\sin\theta=\lambda$
- From the diagram $\tan\theta = \dfrac{y}{D}$
- For small angles (measured in radians), $\sin\theta=\tan\theta=\theta$
- Therefore $\dfrac{\lambda}{d} = \dfrac{y}{D}$
- Rearranging gives

$$\boxed{y = \frac{\lambda D}{d}}$$

Example

- Monochromatic light is incident on a double slit with spacing of 0.50 mm. The bright fringes are spaced 2.5 mm apart on a screen 2.0 m away. What is the wavelength of the light?

$$s = \frac{\lambda D}{d}$$

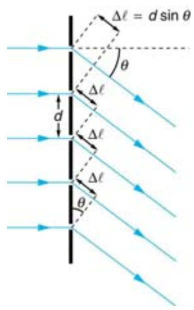
$$\lambda = \frac{sd}{D} = \frac{(2.5\times10^{-3}\,\mathrm{m})(0.5\times10^{-3}\,\mathrm{m})}{2.0\,\mathrm{m}} = 6.25\times10^{-7}\,\mathrm{m}$$

Diffraction Grating

- When light is incident on a diffraction grating it produces interference maxima at angles θ given by

$$n\lambda = d\sin\theta$$

n is the "order" of the maximum (zero for the central maximum, 1 for the first maximum on each side of the center, etc.)



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- The spacing between the slits is small, which makes the angle θ large for a fixed wavelength and n
- Therefore we cannot use the small angle approximation

- A diffraction grating usually gives the lines per mm, N, instead of the distance between the lines
- This value must be converted to spacing, d, to use the diffraction grating equation

$$d = \frac{1}{N}$$

Example

- A diffraction grating having 600 lines per mm is illuminated with a parallel beam of monochromatic light normal to the grating. This produces a second order maximum which is observed at 42.5° to the straight through direction. Calculate the wavelength of the light.

$$n\lambda = d\sin\theta$$

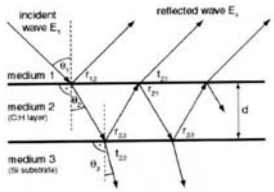
$$d = \frac{1}{600(1000)} = 1.67\times10^{-6}\,\text{m}$$

$$\lambda = \frac{d\sin\theta}{n} = \frac{(1.67\times10^{-6}\,\text{m})\sin(42.5°)}{2}$$

$$\lambda = 5.63\times10^{-7}\,\text{m}$$

Thin Films

- When light is incident on the thin film it is both reflected and refracted at the surface
- The reflected light undergoes a phase shift of π radians



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- The refracted light pass through the thin film a distance of nd where n is the index of refraction and d is the thickness of the film
- The light then partially reflects off the bottom of the film and passes through the film again before reflecting and refracting from the top surface
- Therefore the total path length of the light in the film is $2dn$

- Normally the path difference required for constructive interference is λ
- However, in the case of a thin film, the reflected ray undergoes a phase change of π radians
- This is equivalent to a path difference of $\lambda/2$
- Therefore destructive interference will occur when the path length differs by λ and constructive interference will occur when the path length differs by $\lambda/2$

- Therefore we can state

$$\text{Constructive interference:}\quad 2dn = \left(m + \tfrac{1}{2}\lambda\right)$$
$$\text{Destructive interference:}\quad 2dn = m\lambda$$

Example

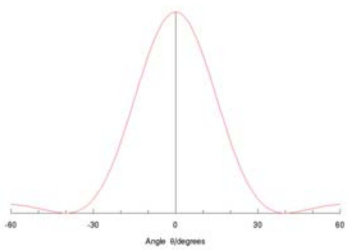
- A film of oil (n=1.45) floats on a layer of water (n=1.33) and is illuminated by light at normal incidence. When viewed from near normal incidence a particular region of the film appears red with an average wavelength of about 650 nm. Calculate the average thickness of the film.

$$2dn = \left(m + \tfrac{1}{2}\right)\lambda$$

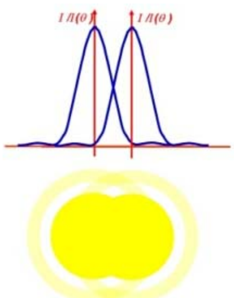
$$d = \frac{\left(m + \tfrac{1}{2}\right)\lambda}{2n} = \frac{\tfrac{1}{2}(650\times10^{-9}\,\mathrm{m})}{2(1.45)} = 1.1\times10^{-7}\,\mathrm{m}$$

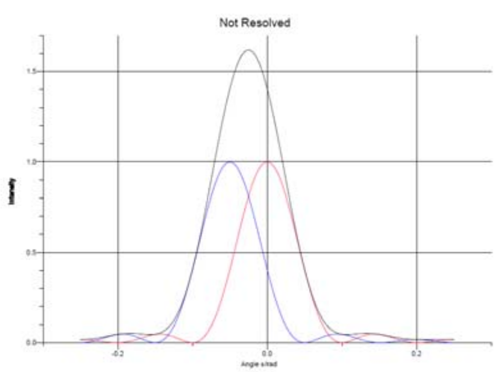
Resolution

- Light from a distant star will, upon passing through a circular aperture, diffract

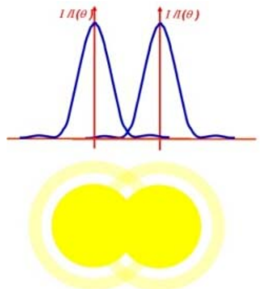


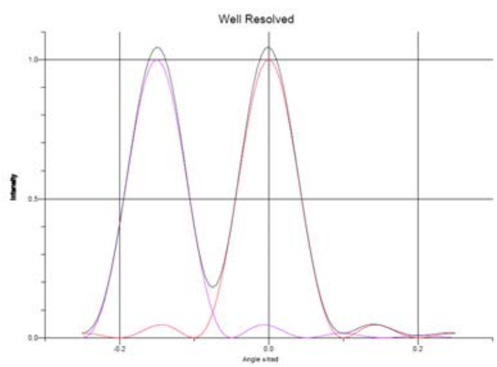
- If the two sources are close, we have a hard time deciding if there are two sources or just one





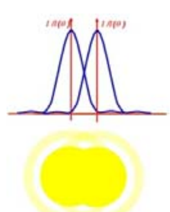
- If the sources are far apart, they are said to be "resolved"
  - We can easily tell that there are two sources

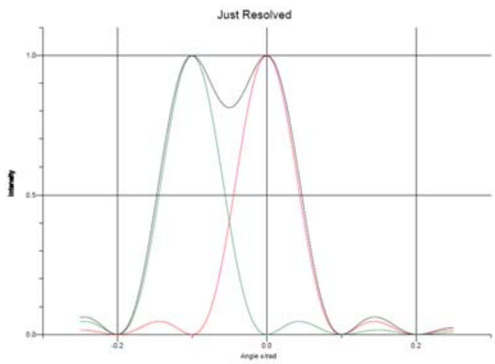




Rayleigh Criterion

- The two sources are just resolved if the central maximum of the diffraction pattern of one source falls on the first minimum of the other





Resolution Equation

- For a linear slit, the first minimum occurs at

$$\theta = \frac{\lambda}{b}$$

- For a circular aperture of diameter b, the first minimum occurs at

$$\theta = 1.22\frac{\lambda}{b}$$

Example

- A student observes two distant point sources of light of wavelength 550 nm. The angular separation as seen by the student is $2.5\text{x}10^{-4}$ radians. Calculate the diameter of the of the student's pupil if the images are just resolved.

$$\theta = 1.22\frac{\lambda}{b}$$

$$b = 1.22\frac{\lambda}{\theta} = \frac{1.22(550\times10^{-9}\,\text{m})}{2.5\times10^{-4}\,\text{radians}} = 2.7\times10^{-3}\,\text{m}$$

Diffraction Gratings

- Diffraction grating are used to separate light of different colors
- The more lines (slits) we have, the better the resolution will be
- The resolvance, R, for a diffraction grating is defined as the ratio of the wavelength of light λ to the smallest difference in wavelength that can be resolved by the grating $\Delta\lambda$

- The resolvance is also equal to mN where N is the total number of slit illuminated by the incident beam and m is the order of the diffraction

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

- The larger the resolvance, the better a device can resolve

Example

- Two lines in the emission spectrum of sodium have wavelengths of 589.0 nm and 589.6 nm. Calculate the number of lines per mm needed by a diffraction grating if the lines are to be resolved in the second order spectrum with a beam of width 0.10 mm.

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

$$N = \frac{\lambda}{\Delta\lambda m} = \frac{589.0\text{nm}}{(586.6 - 589.0\text{nm})2} \quad \text{or} \quad N = \frac{\lambda}{\Delta\lambda m} = \frac{589.6\text{nm}}{(589.6 - 589.0\text{nm})2}$$

$$N = 490.8 \qquad N = 491.3$$

N is the number of lines illuminated by the beam that is 0.1 mm wide. We need the number of lines per mm.

$$\frac{490.8}{.1\text{mm}} = 4908 \text{ lines per mm} \qquad \frac{491.3}{.1\text{mm}} = 4913 \text{ lines per mm}$$

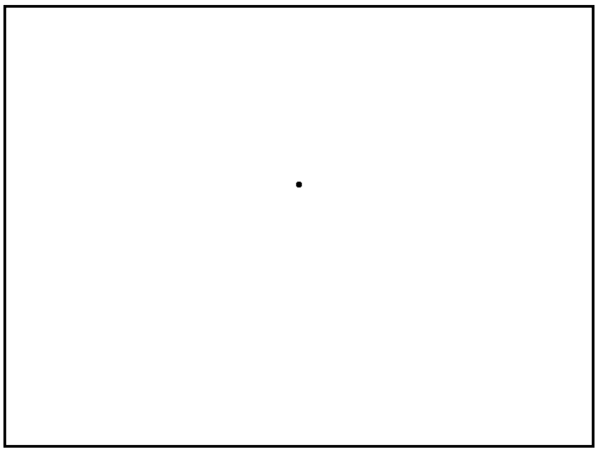
Note: Diffraction gratings are not usually sold with this number of lines per mm. A normal amount would be 5000 lines per mm.

Doppler Effect

- When there is relative motion between a source of waves and an observer, the observed frequency of the waves is different to the frequency of the source of the wave

Stationary Source

- Sound waves are produced at a constant frequency f_0, and the wavefronts propagate symmetrically away from the source at a constant speed v
- The distance between wavefronts is the wavelength.
- All observers will hear the same frequency, which will be equal to the actual frequency of the source



Moving Source

- A source producing the same frequency as before is moving to the right
- The center of each new wavefront is now slightly displaced to the right
- The wavefronts begin to bunch up on the right side (in front of) and spread further apart on the left side (behind) of the source
- An observer in front of the source will hear a higher frequency $f\,' > f_0$, and an observer behind the source will hear a lower frequency $f\,' < f_0$

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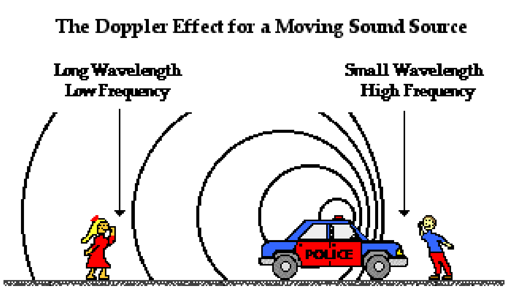


- The frequency that the observer hears is calculated as follows:

$$f' = f\left(\frac{v}{v \pm u_s}\right)$$

Source is moving with speed u_s

Moving Observer

- If the sound source is stationary and the observer is moving the same effect happens
- The apparent frequency increases as the observer moves towards the source
- The apparent frequency decreases as the observer moves away from the source

- The frequency heard by the observer in this case is calculated as follows:

$$f' = f\left(\frac{v \pm u_o}{v}\right)$$

Observer is moving with speed u_o

Light

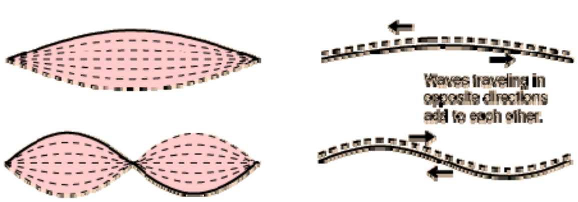
- Light waves are also susceptible to the Doppler effect
- If the source (or observer) is traveling at a speed much less than the speed of light we can approximate the observed change in frequency (or wavelength) as follows:

$$\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$$

Standing (Stationary) Waves

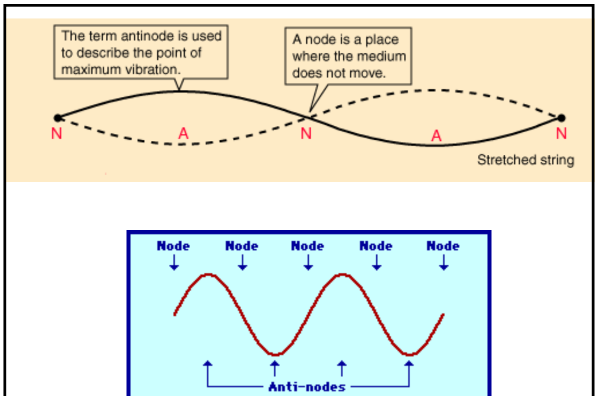
Standing Waves

- Under the right circumstances waves can be formed in which the positions of the crests and the troughs do not change
- Two travelling waves of equal amplitude and equal frequency travelling with the same speed in opposite directions are superposed
- This is a standing wave



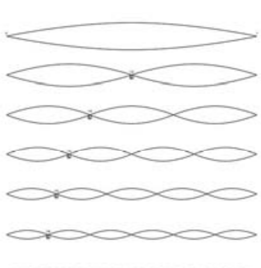
Special Terms

- Node
  - Points where the displacement is ***always*** zero
- Antinode
  - Displacement is a maximum
  - Note: the maximum is not always the same maximum



Harmonics

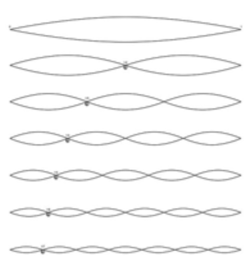
- We can have different modes of vibration or harmonics



- The first mode of vibration has the lowest frequency and is called the first harmonic.
- The next modes of vibration are the second harmonic, third harmonic,…
- Each harmonic is an interval of the fundamental
  – $f_2 = 2f_1$
  – $f_3 = 3f_1$
  – $f_4 = 4f_1$

Standing Waves on a String

- There is a node at each end where the string is attached.



- First harmonic (fundamental)

$$L = \frac{\lambda_1}{2}$$

$$f_1 = \frac{v}{\lambda_1} \qquad \lambda_1 = 2L$$

$$f_1 = \frac{v}{2L}$$

- Second harmonic

$$L = \frac{2\lambda_2}{2}$$

$$f_2 = \frac{v}{\lambda_2} \qquad \lambda_2 = \frac{2L}{2}$$

$$f_2 = \frac{2v}{2L} = 2f_1$$

- Third harmonic

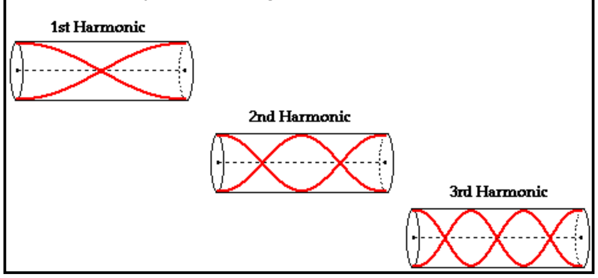
$$L = \frac{3\lambda_3}{2}$$

$$f_3 = \frac{v}{\lambda_3} \qquad \lambda_3 = \frac{2L}{3}$$

$$f_3 = \frac{3v}{2L} = 3f_1$$

Standing Waves in an Open Pipe

- An open pipe (open at both ends) behaves similarly to a string



- First harmonic (fundamental)

$$L = \frac{\lambda_1}{2}$$

$$f_1 = \frac{v}{\lambda_1} \qquad \lambda_1 = 2L$$

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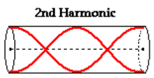
- Second harmonic

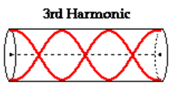


$$L = \frac{2\lambda_2}{2}$$

$$f_2 = \frac{v}{\lambda_2} \qquad \lambda_2 = \frac{2L}{2}$$

$$f_2 = \frac{2v}{2L} = 2f_1$$

- Third harmonic



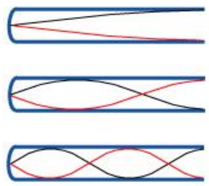
$$L = \frac{3\lambda_3}{2}$$

$$f_3 = \frac{v}{\lambda_3} \qquad \lambda_3 = \frac{2L}{3}$$

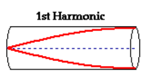
$$f_3 = \frac{3v}{2L} = 3f_1$$

Standing Waves in a Closed Pipe

- A pipe that is closed at one end has a node at the closed end and an antinode at the open end



- First harmonic (fundamental)

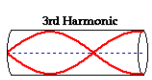


$$L = \frac{\lambda_1}{4}$$

$$f_1 = \frac{v}{\lambda_1} \qquad \lambda_1 = 4L$$

$$f_1 = \frac{v}{4L}$$

- Third harmonic



$$L = \frac{3\lambda_3}{4}$$

$$f_3 = \frac{v}{\lambda_3} \qquad \lambda_3 = \frac{4L}{3}$$

$$f_3 = \frac{3v}{4L} = 3f_1$$

- Fifth harmonic

$$L = \frac{5\lambda_5}{4}$$

$$f_5 = \frac{v}{\lambda_5} \qquad \lambda_5 = \frac{4L}{5}$$

$$f_5 = \frac{5v}{4L} = 5f_1$$