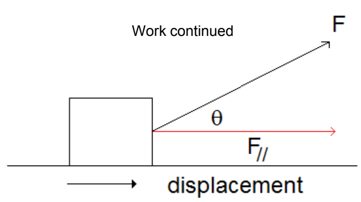
Work, Energy, and Power

Work

- Work done on an object by a constant force is defined to be the product of the magnitude of the displacement times the component of the force parallel to the displacement.

$$W = F_{//} s$$



- So, we can write:

$$W = Fs\cos\theta$$

Units = Nm = <u>Joules</u>

Example #1

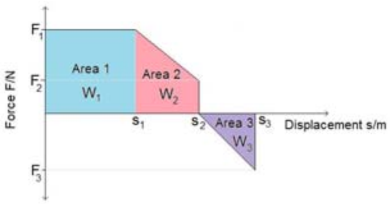
- A 50. kg crate is pulled 40. m along a horizontal floor by a constant force of 100. N exerted by a person at an angle of 37^{o} from the horizontal. The floor is rough and exerts a friction force of 50. N. Determine the work done by each force acting on the crate and the net work done on the crate.

Example #2

- A hiker is carrying a 15.0 kg backpack up a hill with a height of 10.0 m. Determine:
  (a) the work the hiker must do on the backpack.
  (b) the work done by gravity on the backpack.
  (c) the net work on the backpack.
  Note: assume the hiker is travelling at a constant velocity.

One more note about work…

- If the force varies, then the work must be calculated from the area under a force vs. displacement curve.



Energy

- The ability to do work
- A moving object can do work on another object it strikes
  - hammer on nail
  - cannon ball on wall
- This energy of motion is known as **kinetic energy**

Derivation

- Consider a mass, *m*, that is moving in a straight line with an initial speed, v_1. To accelerate it uniformly to speed v_2, a constant net force F_{net} is exerted on it parallel to its motion over a distance, *s*.

$$W_{net} = F_{net} s$$

$$F_{net} = ma$$

$$W_{net} = mas$$

$$v_2^2 = v_1^2 + 2as \qquad a = \frac{v_2^2 - v_1^2}{2s}$$

$$W_{net} = m\left(\frac{v_2^2 - v_1^2}{2s}\right)s$$

$$W_{net} = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

We define the kinetic energy to be $\frac{1}{2}mv^2$

$$\boxed{E_k = \frac{1}{2}mv^2}$$

So…

$$W_{net} = \Delta E_k$$

This is known as the **Work-Energy principle**

Work-Energy Principle

- The work-energy principle tells us that
  - if positive net work, W, is done on a body, its kinetic energy increases by an amount W
  - if negative net work, W, is done on a body, its kinetic energy decreases by an amount W

Example

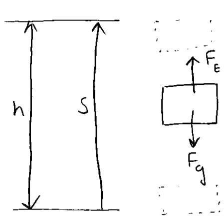
- A 145 g baseball is thrown with a speed of 25 ms^{-1}.
  - What is the kinetic energy of the ball?
  - How much work was done on the ball to make it reach this speed if it started from rest?

Potential Energy

- It is also possible to have potential energy, which is the energy associated with forces that depend on the position or configuration of a body (or bodies) and the surroundings.
  - Wound up clock (work done on spring)
  - Brick held up in the air (gravitational)
  - Propane (chemical)

Gravitational Potential Energy

(most common)



- To lift an object vertically, we must exert a force of at least *mg*.
- To lift without acceleration a height *h* , a person must do work equal to the product of the external force and the distance.

$$W_E = F_E s \cos\theta = mgh$$

Gravity also does work

$$W_g = F_g s \cos\theta = -mgh$$

If we allow the brick to freefall from a height, h, its final velocity will be…

$$v^2 = u^2 + 2as$$

$$(u = 0, \quad a = g, \quad s = h)$$

$$v^2 = 2gh$$

It then would have a kinetic energy of…

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m2gh = mgh$$

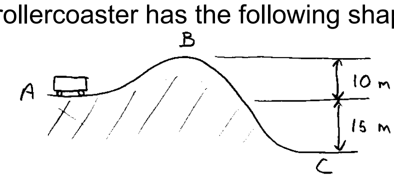
Which is the same amount of work it took to lift it.

Therefore, we define gravitational potential energy as the product of its weight (*mg*) and its height above some reference point.

$$E_p = mgh$$

Example

- A rollercoaster has the following shape:



- The rollercoaster car has a mass of 1000 kg.
  - Determine ΔE_p from A to B; A to C; B to C with A at y=0
  - Determine ΔE_p from A to B; A to C; B to C with C at y=0

Elastic Potential Energy

- A spring can store energy when it is either stretched or compressed
- To hold a spring either stretched or compressed a length *x* from its unstretched length requires a force that is directly proportional to the length of the spring $F_P = kx$

Where *k* is a constant called the "spring constant" and is related to the stiffness of the spring

- The spring stretched or compressed spring exerts a force F_s in the opposite direction acting to return it to its unstretched length (restoring force)

$$F_s = -kx$$

Spring equation
or
Hooke's Law

- To calculate the elastic potential energy of a spring we need to calculate the work required to stretch the spring
- The problem is that the force required to stretch the spring is not constant
- The force is proportional to the distance that the spring is stretched
- Therefore, we must calculate the average force required to stretch the spring

- Since the force varies linearly the average force required to stretch a spring from its unstretched position to a length *x* can be given by $$F_{average} = \frac{0 + kx}{2} = \tfrac{1}{2}kx$$
- The work done in stretching the spring can now be given as

$$W = Fs = F_{average}s = \left(\tfrac{1}{2}kx\right)x = \tfrac{1}{2}kx^2$$

- There the elastic potential energy is

$$\boxed{E_p = \tfrac{1}{2}k\Delta x^2}$$

Conservation of Energy

- The total energy is neither increased nor decreased in any process.
- Energy can be transformed from one form to another, and transferred from one body to another, but the total amount remains constant.

$$\Sigma E_{before} = \Sigma E_{after}$$

Example

- A stone falls from a height of 3.0 m (initial velocity is 0). Calculate the stone's speed when it has fallen to 1.0 m above the ground.

Energy Transformations

- Types of energy
  - Gravitational potential
  - Elastic potential
  - Chemical
  - Nuclear
  - Heat
  - Light

Power

- Average power is defined as
  - the rate at which work is done
  - or
  - the rate at which energy is transformed

$$power = \frac{work}{time} = \frac{Fs}{t}$$

$$\boxed{power = Fv}$$

Units = Js^{-1} = Watt (W)

Example

A 70.0 kg jogger runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m

(a) Calculate the jogger's power output.

(b) How much energy did this require?

Efficiency

- Efficiency is defined as the ratio of work output to energy input.

$$\boxed{efficiency = \frac{useful\ work\ out}{total\ work\ in}}$$

or

$$\boxed{efficiency = \frac{useful\ power\ out}{total\ power\ in}}$$