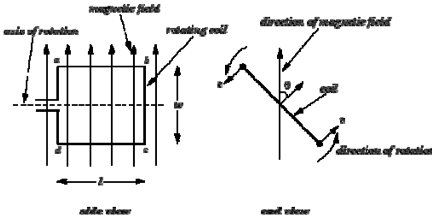
Alternating Current

Alternating Current

- The most important application of the laws of electromagnetic induction was the development of the electric generator.



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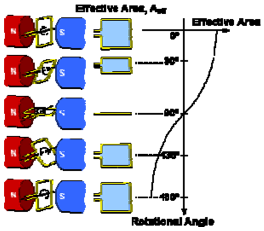
- The flux linkage in the coil varies with time
  - Maximum when coil is perpendicular to the field
  - Zero when the coil is parallel to the field



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We can calculate the emf generated using Faraday's law

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} \qquad \Phi = BA\cos\theta$$

We generate the change in magnetic flux linkage by rotating the loop. That is, we are changing the angle

$$\varepsilon = -NBA\frac{\Delta\cos\theta}{\Delta t} \quad \text{or} \quad \varepsilon = -NBA\frac{d}{dt}(\cos\theta)$$

Differentiating cosθ gives us…

$$\varepsilon = NAB\sin\theta\frac{d\theta}{dt}$$

But $\frac{d\theta}{dt}$ is the angular velocity, ω (rad s^{-1})

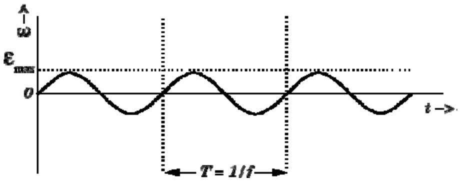
$$\omega = 2\pi f$$

So… $\theta = \omega t = 2\pi ft$

$$\varepsilon = \omega NAB\sin(\omega t)$$

or… $\varepsilon = 2\pi fNAB\sin(2\pi ft)$

- The resulting emf is sinusoidal in nature with the period equal to the rotational period of the generator



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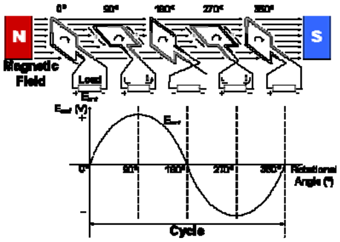
- The emf increases as the flux linkage decreases (and decreases as the flux linkage increases) according to Lenz's law



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Peak Voltage (and Current)

- The value ωNAB represents peak voltage, ε_o
- The peak current can be found as follows:

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_o \sin(\omega t)}{R} = I_o \sin(\omega t)$$

We can also calculate power

$$P = I\varepsilon$$
$$P = I_o \sin(\omega t)\varepsilon_o \sin(\omega t)$$
$$P = I_o\varepsilon_o \sin^2(\omega t)$$

RMS Voltage (and Current)

- It would be convenient to define an average voltage, current, and power.
- For current and voltage, the normally calculated average would be zero.
- So we use a trick:
  - Square the voltage (or current)
  - Find the average of the square
  - Take the square root

- We call this the **root mean squared** (rms) value.

$$\boxed{V_{rms} = \frac{V_o}{\sqrt{2}}} \qquad \boxed{I_{rms} = \frac{I_o}{\sqrt{2}}}$$

- We can also calculate resistance and power with both peak and rms values.

$$\boxed{R=\frac{V_0}{I_0}=\frac{V_{rms}}{I_{rms}}}$$

$$\boxed{P_{\max} = I_0 V_0}$$

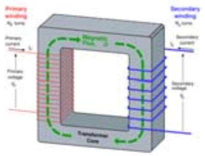
$$P_{rms} = I_{rms} V_{rms}$$

$$\boxed{\overline{P} = \tfrac{1}{2} I_0 V_0}$$

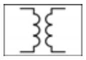
Transformers

- A transformer is a practical application of electromagnetic induction that can be used for increasing or decreasing AC voltages.

- Transformers consist of:
  - Two coils of wire known as the primary and secondary coils
  - Each coil has a laminated soft iron core to reduce eddy currents (increases efficiency)
  - Enclosed on top and bottom with soft iron bars that increase the strength of the magnetic field



www.electricityforum.com/products/trans-s.htm



Schematic Diagram

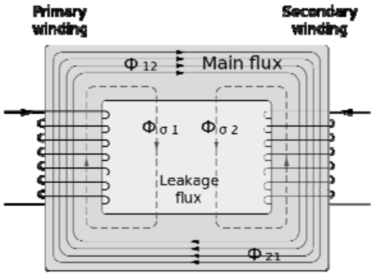
- When a current flows in the primary coil, a magnetic field is produced.
- It grows and “cuts” the secondary coil inducing a current.
- The size of the voltage input/output depends on the number of turns of wire in each coil:

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

- If a transformer is 100% efficient, the power produced in the secondary coil should equal the power input of the primary coil

$$P_p = P_s$$
$$I_p V_p = I_s V_s$$

- In practice, power is lost due to flux leakage (eddy currents) decreasing the maximum possible efficiency



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Example

- A transformer has 50 turns in its primary coil and 1000 turns in its secondary coil. If the input voltage is 110 V, what is the output voltage?

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$$V_s = \frac{V_p N_s}{N_p} = \frac{(110\text{ V})(1000)}{(50)} = 2200\text{ V}$$

Power Transmission

- Electric power is usually transmitted over high voltage (high tension) power lines.
- Copper wire has a resistance and over long runs some energy will be lost to the surroundings as heat.
- A low current (high voltage) minimizes this loss.

Example

- An average of 120 kW is delivered to a suburb 10 km away. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the transmission voltage is:
  - 240 V
  - 24 000 V

240 V

$$P = IV$$

$$I = \frac{P}{V} = \frac{120\text{x}10^3\ \text{W}}{240\ \text{V}} = 500\ \text{A}$$

Power loss:

$$P = I^2 R = (500\ \text{A})^2 (0.40\ \Omega) = 100\ \text{kW}$$

24 000 V

$$P = IV$$

$$I = \frac{P}{V} = \frac{120\text{x}10^3\ \text{W}}{24\,000\ \text{V}} = 5\ \text{A}$$

Power loss:

$$P = I^2R = (5\ \text{A})^2(0.40\ \Omega) = 10\ \text{W}$$