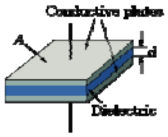
Capacitance

Capacitor

- Any arrangement of two conductors separated from each other by insulating material (dielectric) is called a capacitor



- A capacitor is capable of storing electric charge and electrical energy

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- When a battery is connected to a capacitor a charge of +q accumulates on one plate with an equal charge of –q on the other plate
- A uniform electric field is set up between the two plates
- The amount of charge accumulated on the plates is determined by the capacitance
- Capacitance is defined as the change per unit voltage that can be stored on the capacitor

$$\boxed{C = \frac{q}{V}}$$

Units: Farads, F

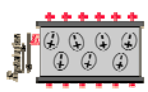
- Capacitance depends on the geometry of the capacitor
- For a parallel plate capacitor

$$\boxed{C = \varepsilon \frac{A}{d}}$$

- Where
  - A is the area of one of the plates
  - d is the separation of the plates
  - ε is the permittivity of the medium
    (for a vacuum, $\varepsilon = \varepsilon_0 = 8.85\text{x}10^{-12}\ \text{Fm}^{-1}$)

The Effect of Dielectric

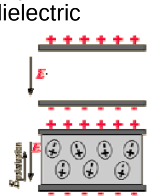
- When a dielectric is placed between charged plates, the polarization of the medium produces an electric field opposing the field of the charges on the plate



- The net electric field is

$$E_{net} = E - E_{polarization}$$

- Consider two capacitors, one with a vacuum and one with dielectric



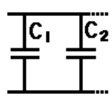
- If the same constant voltage is applied to the capacitors then the potential difference between the plates on each capacitor is constant

- The potential difference is related to the net electric field inside the capacitor

$$E_{net} = \frac{V}{d}$$

- Since the distances are the same, the net electric field must be the same
- This implies that the electric field on the plates in the capacitor with the dielectric is larger than in the one with the vacuum
- That means that the charge, *q*, on the plates is larger with dielectric
- Resulting in a higher capacitance

Capacitors in Parallel



$$q_1 = C_1 V$$
$$q_2 = C_2 V$$

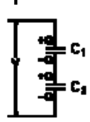
- Total charge on the two capacitors is

$$q = q_1 + q_2 = (C_1 + C_2)V = C_{parallel}V$$

- So we can define total capacitance of capacitors in parallel as

$$\boxed{C_{parallel} = C_1 + C_2 + \ldots}$$

Capacitors in Series



$$q = C_1 V_1 = C_2 V_2$$

- The total voltage V is equal to $V_1 + V_2$

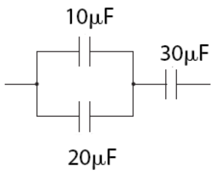
$$V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2} = \frac{q}{C_{series}}$$

- The total capacitance of capacitors in series is thus defined as

$$\boxed{\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots}$$

Example

- Calculate the equivalent capacitance of the following circuit.



$$C = 15\mu F$$

Energy Stored in a Capacitor

- The work done to store an amount of charge in a capacitor is

$$W = \int_0^q V \mathrm{d}q$$

$$W = \int_0^q \frac{q}{C} \mathrm{d}q$$

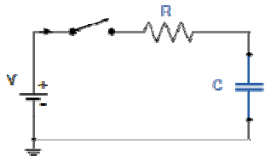
$$W = \frac{q^2}{2C}$$

$$E = W \qquad C = \frac{q}{V}$$

$$E = \frac{q^2}{2C} = \frac{C^2V^2}{2C}$$

$$\boxed{E = \frac{1}{2}CV^2}$$

Charging a Capacitor

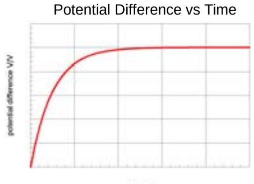


- When the switch is closed current begins to flow and charge on the capacitor plates increases

- As the charge increases the potential difference across the capacitor increases
- Once the capacitor has fully charged, the current stops flowing
- The amount of time required for the capacitor to charge is related to the values of the resistor and the capacitor
- At this point the charge and potential difference are at their maximum and they remain constant until the capacitor is discharged

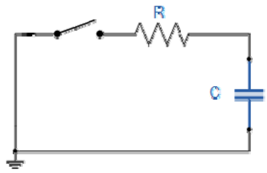
Charge vs Time

Potential Difference vs Time



Current vs Time

Discharging a Capacitor



- When the switch is closed, current begins to flow in the circuit as the capacitor begins to discharge

- The current at a given instant of time is

$$I = -\frac{\mathrm{d}q}{\mathrm{d}t}$$

- Substituting in for current and charge

$$V = IR \qquad q = CV$$

$$V = IR = -R\frac{\mathrm{d}(CV)}{\mathrm{d}t} = -RC\frac{\mathrm{d}V}{\mathrm{d}t}$$

- Solving for V gives

$$V = V_0 e^{-\frac{t}{RC}}$$

Where V_0 is the initial potential difference across the capacitor

- The quantity *RC* is called the **time constant**

$$\boxed{\tau = RC}$$

- A large time constant means that it will take a long time for the capacitor to discharge
- We can write the equation for voltage using this time constant

$$\boxed{V = V_0 e^{-\frac{t}{\tau}}}$$

- Recognizing that $q=CV$, we can calculate the charge over time as follows

$$\boxed{q = q_0 e^{-\frac{t}{\tau}}}$$

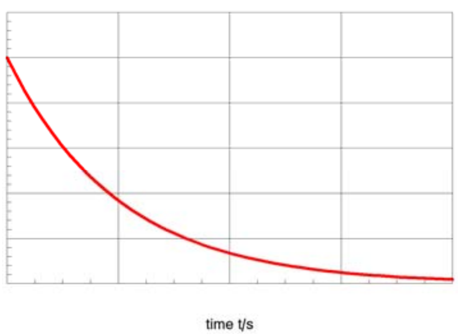
Where q_0 is the initial charge given by $q_0 = CV_0$

- Similarly, since $V=IR$, we can calculate the current

$$\boxed{I = I_0 e^{-\frac{t}{\tau}}}$$

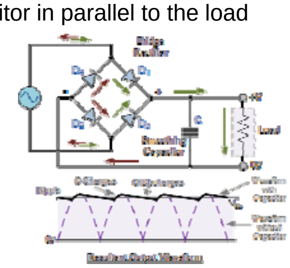
Where I_0 is the initial charge given by $I_0 = \frac{q_0}{RC} = \frac{q_0}{\tau}$

- Graphing all of these equations gives the same shape



Capacitors in Rectification

- The output of a diode bridge rectifier may be processed further make is smoother by adding a capacitor in parallel to the load



Why is the output smoother?

- As the input voltage increases, the capacitor charges
- When the input voltage starts to decrease, the capacitor discharges through the load causing its voltage to decrease
- When the input voltage is higher than the capacitor voltage, the capacitor once again begins to charge
- The process continues giving us an almost smooth DC current