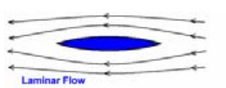
Fluid Dynamics

Streamline (laminar) flow

- Each particle of the fluid follows a smooth path (streamline) and these paths do not cross one another

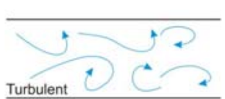
quoteimg.com



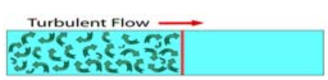
ffden-2.phys.uaf.edu

Turbulent flow

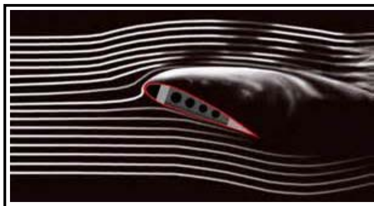
- Characterized by erratic, small, whirlpool-like circles called eddy currents



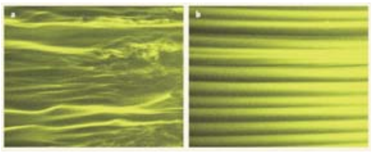
quoteimg.com



www.globalspec.com



www.hobiecat.com

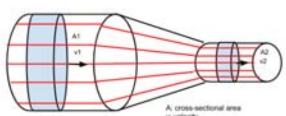


indocheatvip.com

esfsciencenew.wordpress.com

Flow Rate

- Consider the steady laminar flow of fluid through an enclosed pipe as shown



calengineeringabroad.wordpress.com

- The mass flow rate must be equal at both ends of the pipe

- The volume of fluid passing through A_1 in time Δt is $A_1 \Delta l_1$, where Δl_1 is the distance the fluid moves in time Δt
- The mass flow rate at point 1 is therefore

$$\frac{m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

- This must equal the mass flow rate at point 2

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

- This is called the equation of continuity

- Assuming that the fluid is incompressible (ρ does not change with pressure)
  - Valid assumption for liquids under most circumstances

$$A_1 v_1 = A_2 v_2$$

or

$$\boxed{Av = \text{constant}}$$

- The product Av represents the volume flow rate (or just flow rate)

Example

In humans, blood flows from the heart into the aorta, then arteries, and eventually into a myriad of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about 1.2 cm and the blood passes through it at a speed of about 40 cms^{-1}. A typical capillary has a radius of about 4x10^{-4} cm and blood flows through with a speed of about 5x10^{-4} ms^{-1}. Estimate the number of capillaries in the human body.

$$A_1 v_1 = A_2 v_2$$

$$\pi r_{aorta}^2 v_1 = N \pi r_{capillaries}^2 v_2$$

$$N = \frac{r_{aorta}^2 v_1}{r_{capillaries}^2 v_2}$$

$$N = \frac{(1.2\times10^{-2}\,\text{m})^2 (0.4\text{ms}^{-1})}{(4\times10^{-6}\,\text{m})^2 (5\times10^{-4}\,\text{ms}^{-1})}$$

$$N = 7\times10^9$$

Bernoulli's Principle

- Daniel Bernoulli (1700-1782) worked out a principle concerning fluids in motion
- Where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high
- Bernoulli developed an equation that expresses this principle quantitatively

Bernoulli's Equation

$$\tfrac{1}{2}\rho v^2 + \rho g z + p = \text{constant}$$

ρ – fluid density
v – speed of fluid
g – gravitational field strength
z – the height above a chosen level
p – the pressure at the height z

- Bernoulli's equation is an expression of conservation of energy

Example

- Water circulates through a house in a hot water system. The water enters the house with a speed of 0.50 ms^{-1} through a 4.0 cm diameter pipe with a pressure of 3.0×10^5 Pa. Calculate the flow rate and pressure in a 1.0 cm diameter pipe on the second floor 5.0 m above. Assume the pipes do not divide into branches.

- Calculate flow rate on second floor

$$A_1 v_1 = A_2 v_2$$
$$v_2 = \frac{A_1 v_1}{A_2}$$
$$v_2 = \frac{\pi (2\times 10^{-2}\,\mathrm{m})^2 (0.5\,\mathrm{ms}^{-1})}{\pi (0.5\times 10^{-2})^2}$$
$$v_2 = 8.0\,\mathrm{ms}^{-1}$$

- Calculate pressure on second floor

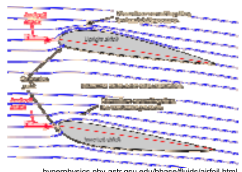
$$\tfrac{1}{2}\rho v_1^2 + \rho g z_1 + p_1 = \tfrac{1}{2}\rho v_2^2 + \rho g z_2 + p_2$$
$$p_2 = \tfrac{1}{2}\rho (v_1^2 - v_2^2) - \rho g (z_1 - z_2) + p_1$$
$$p_2 = \tfrac{1}{2}(1000\,\mathrm{kgm}^3)\left((0.5\,\mathrm{ms}^{-2})^2 - (8\,\mathrm{ms}^{-2})^2\right) - (1000\,\mathrm{kgm}^3)(9.81\,\mathrm{ms}^{-2})(0-5\,\mathrm{m}) + 3\times 10^5\,\mathrm{Pa}$$
$$p_2 = 3.2\times 10^5\,\mathrm{Pa}$$

Applications of Bernoulli's Principle

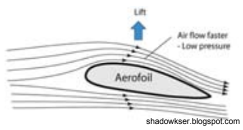
- Airplane wings and dynamic lift
- Venturi tubes
- Pitot static tubes
- Baseball
- Flow out of a container
- Lack of blood to the brain

Airplane Wings

- Simplistically, the velocity of the air going over the top of the wing is greater than under the bottom
- This causes a pressure difference due to Bernoulli's principle and thus lift



hyperphysics.phy-astr.gsu.edu/hbase/fluids/airfoil.html

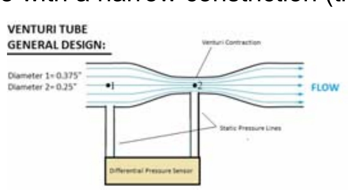


shadowkser.blogspot.com

- Realistically, the pressure varies along curved streamlines and therefore Bernoulli's equation must be applied separately at every point on each streamline
- Lift occurs because the streamlines follow the curvature of the wing
- While it is not necessary to consider friction to describe lift, it is because of friction that the streamlines take the shape of the wing

Venturi Tubes

- A pipe with a narrow constriction (throat)

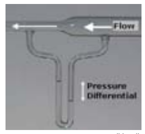


edge.rit.edu

- The flowing air speeds up as it passes through this constriction, so the pressure is lower in the throat

- A venturi meter is used to measure the flow speed of gases and liquids

triad-measurement.com/article_22_Venturi-Tubes.cfm



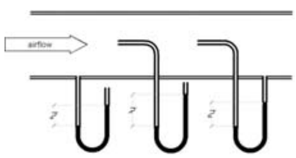
en.wikipedia.org

www.triflotech.com/venturi-tubes.html

Dtom, commons.wikimedia.org/wiki/File:Venturi_tube.jpg

Pitot Tubes

- Used for measuring the velocity of a fluid
- It measures the difference between static, total, and dynamic pressure



engineeringtoolbox.com

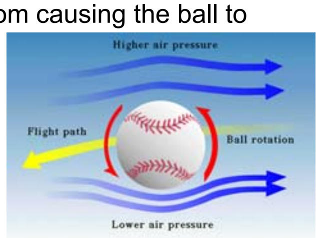
www.m0a.com

aerowiki-info.blogspot.com

Baseball

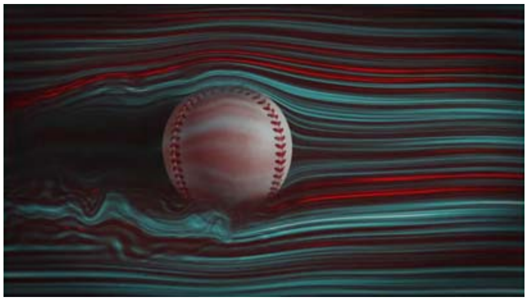
- When throwing a curveball, the pitcher puts a spin on the ball as it is leaving his hand
- The ball drags a thin layer of air with it ("boundary layer") as it travels

- Friction provided by the stitches of the baseball causes a thin layer of air to move around the spinning ball in such a way that air pressure on top of the ball is greater than on the bottom causing the ball to curve downward



w3.shorecrest.org/~Lisa_Peck/Physics/syllabus/phases/gases/gaswp05/justin1/home.html

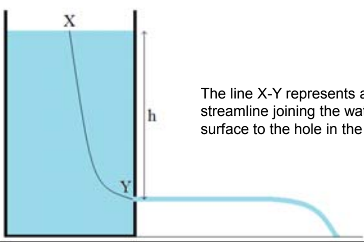
- Consequently, a spinning baseball has more air turbulence on top of the ball, which produces a slower air speed over the ball
- At the same time, air moving under the ball accelerates and moves faster, producing less pressure on the bottom of the ball
- The ball moves downward faster than would normally be expected because of this.



diamondkinetics.com/bernoullis-principle-applied-to-baseball/

Flow out of a container

- Bernoulli's equation can be used to calculate the velocity a liquid flowing out of a hole at the bottom of a container



The line X-Y represents a streamline joining the water at the surface to the hole in the side.

- Both points on the streamline are open to the atmosphere so they will be at the same pressure (atmospheric pressure)
- If the diameter of the hole is much smaller than the opening at the top, then the velocity at the top will be approximately zero
- The Bernoulli equation will be

$$\rho g h = \tfrac{1}{2}\rho v_y^2$$

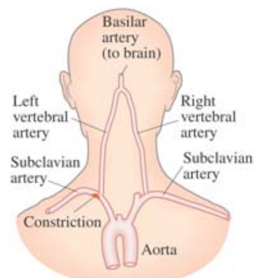
$$v_y = \sqrt{2gh}$$

- This result is called Torricelli's theorem

Lack of Blood to the Brain

- Bernoulli's principle is used to explain a TIA (transient ischemic attack – temporary lack of blood supply to the brain)
- Blood normally flows to the brain through two vertebral arteries (one on either side of the neck)
- These arteries are connected to the subclavian arteries

- If there is a blockage in the subclavian artery on one side, then the blood velocity on that side will increase
- This will result in lower pressure at the vertebral artery

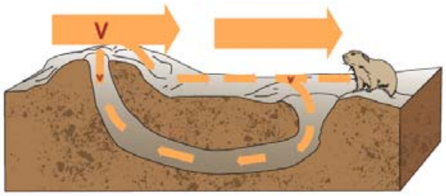


- Thus blood flowing up the "good" side may be diverted down into the other vertebral artery

Underground Burrows

- If animals that live underground are to avoid suffocation, the air must circulate in their burrows
- The burrow must have at least two different entrances
- The speed of the air at the two entrances will usually be slightly different resulting in a pressure difference at each opening

- This will force air to flow through the burrow (following Bernoulli's principle)



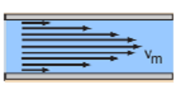
www.asknature.org/strategy/e27b89ebcdec8c9b5b2cd9ac84b8f8a0

- If one entrance is higher than the other, then the effect is enhance (since wind speed tends to increase with height)

Viscosity

- Real fluids have a certain amount of internal friction called viscosity
- In a viscous fluid in laminar flow, each layer impedes the motion of its neighboring layers

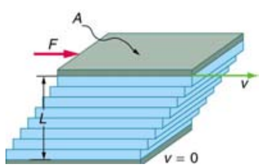
- In a pipe, the layers adjacent to the walls are stationary while the layers in the center travel with greatest speed



hyperphysics.phy-astr.gsu.edu/hbase/pfric.html

- Viscosity is highly dependent on temperature
  - Liquids become less viscous at higher temperatures
  - Gasses become more viscous at higher temperatures

- Viscosity is expressed quantitatively by a coefficient of viscosity, η
- The coefficient of viscosity is calculated determining the force necessary to drag a fluid between two plates



cnx.org/contents/pCk_wk3i@5/Viscosity-and-Laminar-Flow-Poi

$$\eta = \frac{FL}{vA}$$

Units: Pa s

Stoke's Law

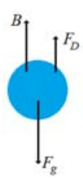
- As a sphere falls through a viscous liquid it flows around it
- If we knew the precise velocities near the sphere we could calculate the total viscous force by integrating over the sphere
- This was done by Sir George Gabriel Stokes (born in Ireland) in the 1940s.

- The drag force F_D on a sphere of radius r moving through a fluid of viscosity η at speed v is given by:

$$F_D = 6\pi\eta r v$$

Example

- A stainless steel (ρ=8000 kgm^{-3}) ball of radius 1 cm is dropped into olive oil (ρ=800 kgm^{-3}, η=0.1 Pa s). Calculate the terminal velocity of the ball.



$$\sum F = F_g - B - F_D$$

$$= mg - \rho_f V_f g - 6\pi\eta r v$$

$$m_b = \rho_b V \qquad V = \frac{4\pi r^3}{3}$$

- At terminal velocity the net force on the ball is zero.

$$0 = \rho_b V g - \rho_f V g - 6\pi\eta r v$$

$$v = \frac{Vg(\rho_b - \rho_f)}{6\pi\eta r}$$

$$= \frac{4\pi r^3 g(\rho_b - \rho_f)}{(3)6\pi\eta r}$$

$$v = \frac{2r^2 g(\rho_b - \rho_f)}{9\eta}$$

$$v = \frac{2(0.01\,\text{m})^2(9.81\,\text{ms}^{-2})(8000\,\text{kgm}^{-3} - 800\,\text{kgm}^{-3})}{9(0.1\,\text{Pa s})}$$

$$v = 16\,\text{ms}^{-1}$$

Turbulence

- At low velocities fluids flow steadily and in layers that do not mix
- As the velocity increases or objects project into the fluid it becomes turbulent
- It is not easy to predict when the rate of flow is sufficiently high to cause the onset of turbulence

Reynolds Number

- The Reynolds number is a dimensionless quantity that is used to help predict the transition from laminar to turbulent flow
- The concept was introduced by George Gabriel Stokes in 1851, but named after Osborne Reynolds (1842–1912), who popularized its use in 1883

- For a fluid flowing with speed v in a pipe of radius r, the Reynolds number is defined as:

$$R = \frac{v r \rho}{\eta}$$

- We have turbulent flow if this number exceeds about 1000

Example

- Air of density 1.2 kgm^{-3} flows at a speed of 2.1 ms^{-1} through a pipe of radius 5.0 mm. The viscosity of the air is 1.8×10^{-5} Pa s.
  - Show that the flow is laminar.
  - Above what speed would the flow become turbulent?

$$R = \frac{v r \rho}{\eta} = \frac{(2.1\,\mathrm{ms}^{-1})(5.0\times10^{-3}\,\mathrm{m})(1.2\,\mathrm{kgm}^{-3})}{1.8\times10^{-5}\,\mathrm{Pa\,s}}$$
$$= 700$$

- R < 1000 so flow is laminar
- Minimum speed for turbulent flow would be for R=1000

$$v = \frac{R\eta}{r\rho} = \frac{(1000)(1.8\times10^{-5}\,\mathrm{Pa\,s})}{(5.0\times10^{-3}\,\mathrm{m})(1.2\,\mathrm{kgm}^{-3})}$$
$$= 3\,\mathrm{ms}^{-1}$$