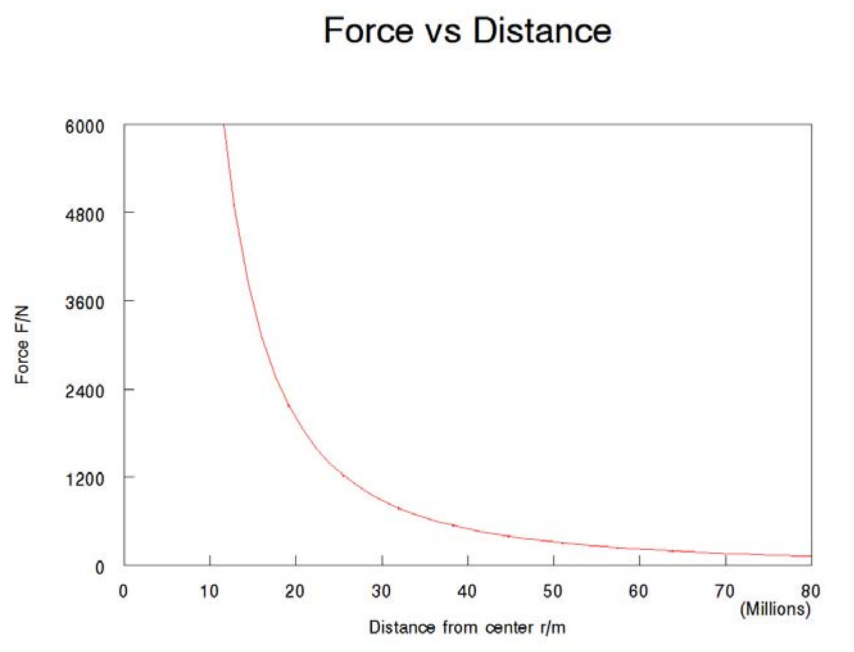
# Motion in a Gravitational Field

## Another Look at Gravitational Potential Energy

- We previously noted that it took work to lift an object off the surface of the earth.
- In more general terms, it takes work to separate two objects.
- Work can be calculated be finding the area under the Force vs Distance curve.
- The work done is equivalent to the Gravitational Potential Energy.




- Force $F = G\frac{m_1 m_2}{r^2}$

- Potential Energy $E_p = -G\frac{m_1 m_2}{r}$

  - Force at an infinite distance is 0
  - Therefore, potential energy at an infinite distance must be 0
  - Potential energy decreases as the distance decreases, therefore $E_p$ must be negative

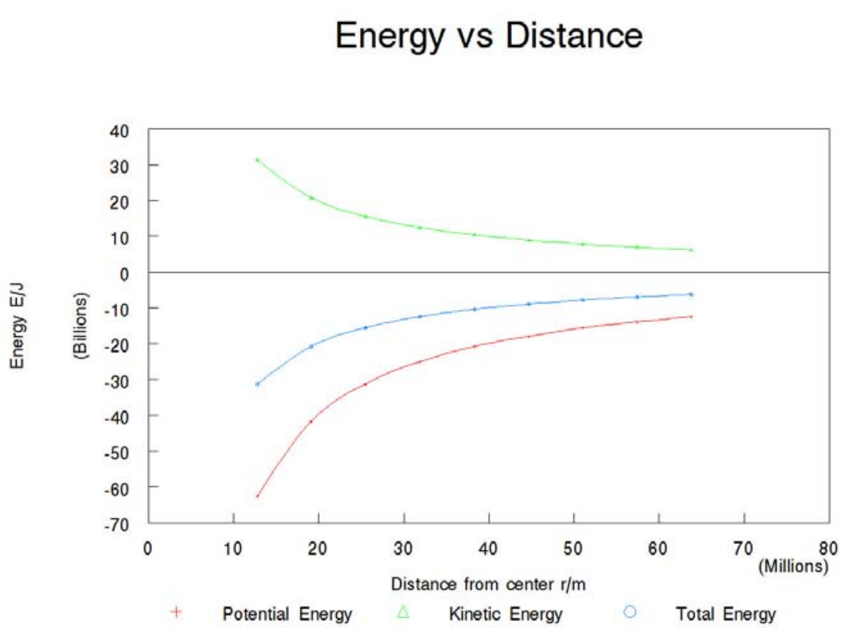
# Total Energy

- The total energy of a satellite is its kinetic energy plus its potential energy

$$E = \frac{1}{2}mv^2 - G\frac{m_1 m_2}{r}$$

- After some manipulation, we can see that…

$$E_k = G\frac{m_1 m_2}{2r} \quad \text{and} \quad E = -G\frac{m_1 m_2}{2r}$$



# Gravitational Potential

- Gravitational potential ,*V*, is a field
  - Defined at every point in space
  - Scalar quantity
- Work done per unit mass in bringing a small point mass $m_p$ from infinity to point P.

- If the work done is *W*, then the gravitational potential is the ratio of work done to the mass $m_p$

$$V = \frac{W}{m_p}$$

- The gravitational potential due to a single mass *m* a distance *r* from the center of *m* is

$$\boxed{V = -\frac{Gm}{r}}$$

# Potential Difference

- It is sometimes convenient to determine the change in gravitational potential or the potential difference between two points in the gravitational field.

$$\Delta V = \frac{\Delta W}{m}$$

But the change in work is the change in energy (potential in this case), so…

$$\Delta V = \frac{\Delta E_p}{m}$$

# Gravitational Field Strength (again)

- It also should be noted that we can calculate the gravitational field strength in terms of gravitational potential

$$g = -\frac{\Delta V}{r}$$

# Escape Velocity

- How fast do you have to go to escape the gravitational field of a planet (or any massive object)?
- The total energy of a moving object, *m*, near a large stationary mass, *M*, is

$$E = \frac{1}{2}mv^2 - G\frac{mM}{r}$$

- At a long distance away (∞), then the mass, m, should only have kinetic energy
- Conservation of energy tells us that these two energies should be equal to each other
- Therefore, for the total energy, if
  - E>0: mass escapes and never returns
  - E<0: mass moves out a certain distance but is pulled back
  - E=0: mass just barely escapes
- We use this third case to find the escape velocity

$$\frac{1}{2}mv^2 - G\frac{mM}{r} = 0$$

$$v = \sqrt{\frac{2GM}{r}}$$

- This is the escape velocity for any planet
- Using gravitational field strength, g

$$g = G\frac{M}{r^2}$$

$$v = \sqrt{2gr}$$

# Orbital Motion

- Kepler deduced that the planets orbited the sun in elliptical paths from observations made by Tycho Brahe
- Newton's law of universal gravitation and his second law of motion provide a theoretical understanding of Kepler's conclusions

- Consider a planet (mass=*m*) in a circular orbit of radius *r*, around the sun (mass=*M*)

$$F = G\frac{mM}{r^2}$$

$$G\frac{mM}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

- This also applies to satellites orbiting Earth (*M* would be the mass of Earth)

- What if we wanted to use period instead?

$$G\frac{mM}{r^2} = \frac{m4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

- Kepler had experimentally discovered this relationship before Newton
- It is called Kepler’s Third Law of Planetary Motion

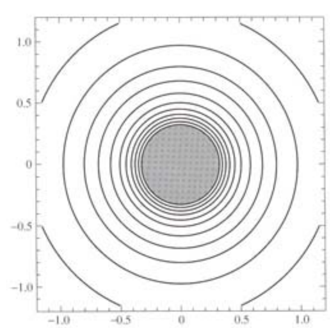
$$T \propto \sqrt{r^3}$$

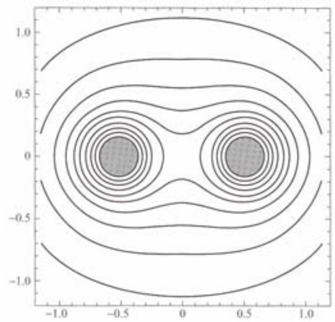
# Weightlessness

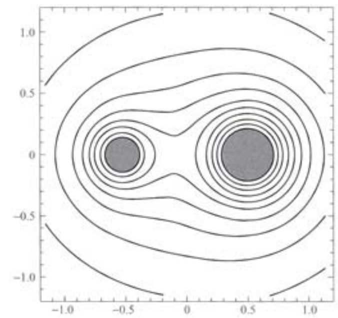
- Why does an astronaut in a spaceship orbiting the Earth feel weightless?
- There is a gravitational force acting on the spaceship (and thus the astronaut)
- BUT
- The astronaut AND the spaceship are free falling (it just happens to be moving in a circle) and so there are no reaction forces from the floor of the spaceship

# Equipotential Surfaces

- An equipotential surface is a graphical way of showing where the gravitational potential is the same

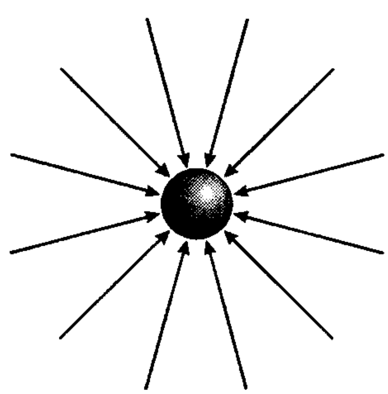






# Gravitational Field Lines

- Gravitational field lines show the direction of the gravitational field



- Equipotential lines and field lines are perpendicular to each other.