Gravitational Force, Potential Energy, and Potential Worksheet

① $\frac{G \cancel{m_1} m_2}{r^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{r}}$

$v = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\ Nm^2 kg^{-2})(5.98 \times 10^{24}\ kg)}{(6.38 \times 10^6\ m + 500 \times 10^3\ m)}}$

$\underline{v = 7.61 \times 10^3\ ms^{-1}}$

$\frac{G \cancel{m_1} m_2}{r^2} = \cancel{m_1} \frac{4\pi^2 r}{T^2}$

$T = \sqrt{\frac{4\pi^2 r^3}{Gm}} = \sqrt{\frac{4\pi^2 (6.38 \times 10^6\ m + 500 \times 10^3\ m)^3}{(6.67 \times 10^{-11}\ Nm^2 kg^{-2})(5.98 \times 10^{24}\ kg)}}$

$T = \underline{5680\ s} = \underline{94.6\ min}$

② $\frac{G \cancel{m_1} m_2}{r^2} = \frac{\cancel{m_1} 4\pi^2 r}{T^2}$

$r = \sqrt[3]{\frac{GmT^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67 \times 10^{-11}\ Nm^2 kg^{-2})(5.98 \times 10^{24}\ kg)(24 \times 3600)^2}{4\pi^2}}$

$r = 4.23 \times 10^7\ m$ (from the center of the Earth)

$r = 4.23 \times 10^7\ m - 6.38 \times 10^6\ m = \underline{3.59 \times 10^7\ m}$ from the surface

③ $E_p = -\frac{G m_1 m_2}{r} = -\frac{(6.67 \times 10^{-11}\ Nm^2 kg^{-2})(5.98 \times 10^{24}\ kg)(7.35 \times 10^{22}\ kg)}{(384 \times 10^6\ m)}$

$= \underline{-7.63 \times 10^{28}\ J}$

④ $V = -\frac{Gm}{r} = -\frac{(6.67 \times 10^{-11}\ Nm^2 kg^{-2})(5.98 \times 10^{24}\ kg)}{(384 \times 10^6\ m)}$

$= \underline{-1.04 \times 10^6\ J\ kg^{-1}}$

(5) $E_p = -\frac{Gm_1 m_2}{r} = \frac{-(6.67\times10^{-11}\,Nm^2kg^{-2})(5.98\times10^{24}\,kg)(500\,kg)}{5(6.38\times10^{6}\,m)}$

$= \underline{-6.25\times10^{9}\ J}$

$V = -\frac{Gm}{r} = \frac{-(6.67\times10^{-11}\,Nm^2kg^{-2})(5.98\times10^{24}\,kg)}{5(6.38\times10^{6}\,m)}$

$= \underline{-1.25\times10^{7}\ J\,kg^{-1}}$

(6) $E_T = E_k + E_p$

$= \frac{1}{2}mv^2 + -\frac{GmM}{r}$

$F = \frac{G\cancel{m}M}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$

$v^2 = \frac{GM}{r}$

$E_T = \frac{1}{2}m\left(\frac{GM}{r}\right) - \frac{GmM}{r}$

$\therefore E_T = \underline{-\frac{GmM}{2r}}$

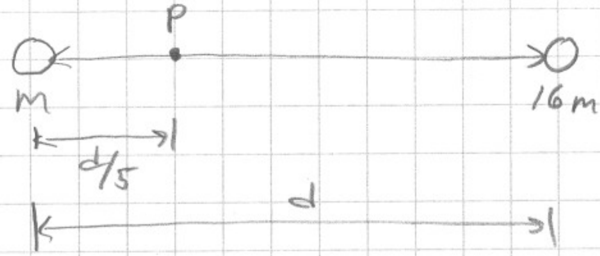
(7) $\frac{1}{2}\cancel{m}v^2 = \frac{G\cancel{m}M}{r}$ for escape velocity.

$v = \sqrt{\frac{2GM}{r}}$

$g = \frac{F}{m} = \frac{G\cancel{m}M}{\cancel{m}r^2}$

$\therefore \underline{v = \sqrt{2gr}}$

(8) (a)



$$g = \frac{F}{m}$$

so if $F = 0$ then $g = 0$

$$F_{\text{left of } P} = \frac{G\, m_1 (m)}{\left(\frac{d}{5}\right)^2} \qquad F_{\text{right of } P} = \frac{G\, m_1 (16m)}{\left(\frac{4d}{5}\right)^2}$$

For the force at P to be zero, these two forces must be equal.

$$\frac{\cancel{G}\,\cancel{m_1}\,\cancel{m}}{\left(\frac{\cancel{d}}{\cancel{5}}\right)^2} = \frac{\cancel{G}\,\cancel{m_1}\,(16\cancel{m})}{\left(\frac{4\cancel{d}}{\cancel{5}}\right)^2}$$

$$1 = 1$$

(b)
$$V = -\frac{Gm}{\frac{d}{5}} + -\frac{G(\cancel{16}^{4}m)}{\frac{4d}{5}}$$

add the potential from both masses.

$$= -\frac{5Gm}{d} - \frac{G\,4m(5)}{d}$$

$$= -\frac{25Gm}{d}$$

(9)
$$F = \frac{G\,\cancel{m}\,M}{r^2} = \frac{\cancel{m}\,4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

(10)
$$F = \frac{GmM}{R^2} = \frac{m\,4\pi^2 R}{T^2} \qquad g = \frac{F}{m} = \frac{GM}{R^2}$$

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$$