Modeling a Gas

Avagadro Constant

- One mole of a substance is a quantity of the substance that contains a number of particles equal to the Avagadro constant and whose mass in grams is equal to the molar mass of the substance.

$$N_A = 6.02 \times 10^{23}$$

Pressure

- Normal force applied per unit area

$$p = \frac{F}{A}$$

Units: Nm^{-2} or Pa

Non-SI Unit: atm (1 atm = 101.3 kPa)

Ideal Gas

- An ideal gas is a theoretical model of a gas
- It cannot be liquefied or solidified
- Real gasses may be approximated by an ideal gas when the density is low
- Real gasses will not behave as ideal gasses at low temperatures

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Gas Laws

- The state of a gas is determined when we know the values of the pressure, volume, temperature, and the number of moles present
- Pressure, volume, temperature, and number of moles are related to each other
- The equation relating them is called the equation of state

P-V (constant T)
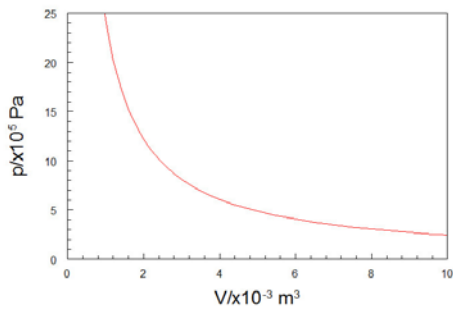
- Linearizing the curve gives us
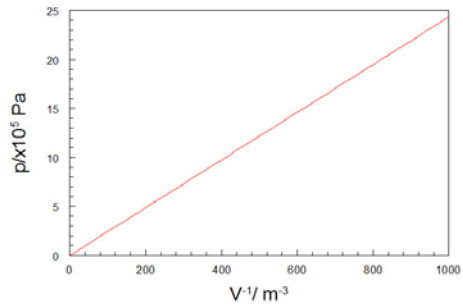
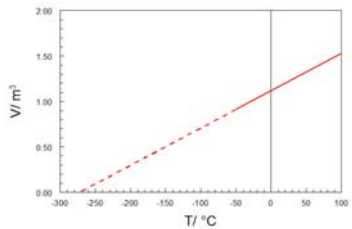- Pressure is inversely proportional to volume

$$p \propto \frac{1}{V}$$

- Discovered independently by Robert Boyle of Ireland in 1662 and Edme Mariotte of France in 1676
- In Great Britain, America, Australia, the West Indies and other remnants of the British Empire it is called Boyle's law, while in Continental Europe and other places it is called Mariotte's law.
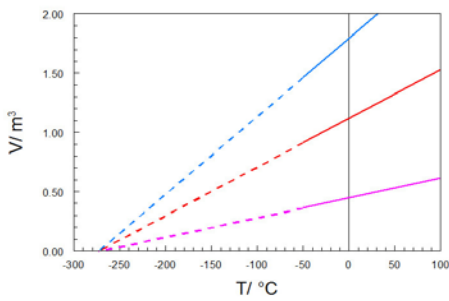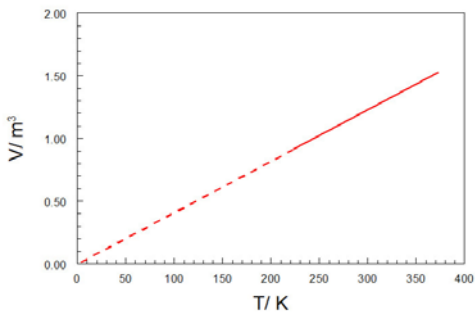
V-T (constant P)
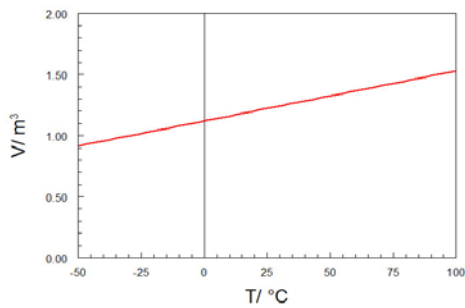
- A real gas will solidify at very low temperatures so there is a physical limit to our measuring abilities
- If the line is extended, it will have an intercept at T = -273.15 °C

- Different amounts of gas (or pressure) all converge at the same point

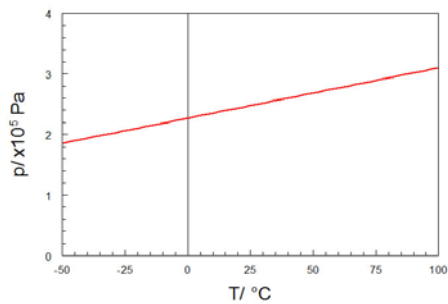- When temperature is expressed in kelvin, the line intersects the origin

- Volume is directly proportional to temperature (in Kelvin)
$$V \propto T$$
- Discovered by French scientist Gay-Lussac and English scientist John Dalton independently
- Each published his results around 1802, with Gay-Lussac's experimental work being both more thorough and more precise than that of Dalton

- Credit for this discovery typically goes to neither Dalton nor Gay-Lussac, but instead to French scientist Jacques Charles who had done some initial work on the thermal expansion of gases in 1787
- Charles never published the results of his experiments

p-T (constant V)

- Extending the line it can be seen that it intercepts at T= -273.15 °C
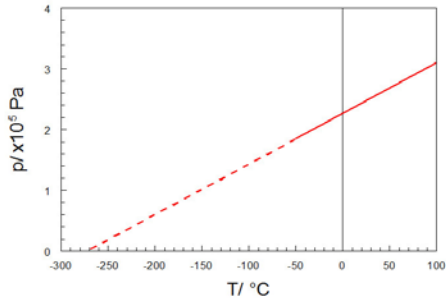
- Different volumes of gas all converge at the same point

- When temperature is expressed in kelvin, the line goes through the origin

- Pressure is directly proportional to temperature when temperature is expressed in kelvin
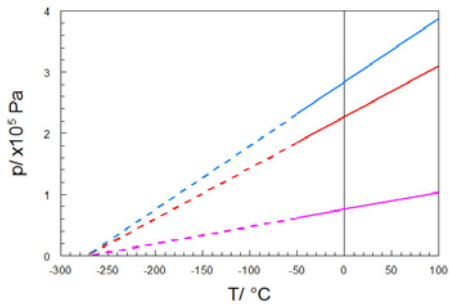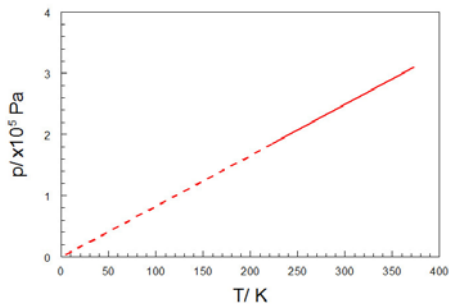
$$p \propto T$$

- French physicist Guillaume Amontons investigated the relationship between pressure and temperature in gases
- Toward the end of the 1600s, he built a thermometer based on the fact that the pressure of a gas is directly proportional to its temperature

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- Amontons' experiments lead to the inference of an absolute lowest temperature
- Gay-Lussac also did experiments with pressure and temperature
- He was able to show the precise mathematical relationship between the two

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Equation of State of an Ideal Gas

- Combining the three gas laws gives

$$\frac{pV}{T} = \text{constant}$$

- Experiments show that the constant is proportional to the number of moles of the gas

$$\frac{pV}{T} = n \times \text{constant}$$

- Further experiments allow us to determine the constant (called the gas constant)
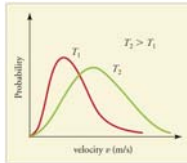
$$R = 8.31\,\mathrm{JK^{-1}mol^{-1}}$$

- The final equation of state is therefore written as

$$\boxed{pV = nRT}$$

Boltzmann Equation

- The molecules in a gas move randomly with a range of speeds
- The distribution of speeds depends on temperature

cnx.org/contents/bb091009-f21f-47a2-954b-3b79871974b3@4/Kinetic-Theory-Atomic-and-Mole

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Kinetic Model of an Ideal Gas

- Gases are made up of molecules that are point masses and very far apart (the space between each individual molecule is many orders of magnitude greater than the diameter of the molecule).
- Molecules are in constant random motion with no general pattern governing either the magnitude or direction of the velocity.

- The movement of molecules is governed by Newton's Laws and there are no forces between any of the molecules except in a collision.
- Molecular collisions with each other and the container walls are perfectly elastic and the duration of the collision is negligible compared to the time between collisions.

Kinetic Model Explanations

- We can use the kinetic model of an ideal gas to explain
  - How the gas exerts pressure in the container
  - How temperature is related to the movement of the molecules
  - Determine the internal energy of the molecules

Pressure

- Consider a molecule colliding with a wall
- Before the collision the molecule has a momentum $p_b = mv$
- After the collision the molecule has a momentum $p_a = -mv$
- The change in momentum is $2mv$
- The change in momentum is caused by a force

- Each molecule hitting the container wall is exerting a force
- The gas pressure in the container is the total force of all of these colliding molecules over the area of the container

$$p = \frac{1}{3}\frac{Nm\bar{v}^2}{V}$$

N = number of molecules

m = mass of molecule

$\bar{v}$ = average velocity

V = volume of container

Temperature

- The equation for pressure can be rewritten as

$$p = \frac{\frac{2}{3}N(\frac{1}{2}m\bar{v}^2)}{V}$$

- The quantity $\frac{1}{2}m\bar{v}^2$ is the average kinetic energy $(\bar{E}_k)$ of the molecules in the gas
- Substituting into the ideal gas equation gives

$$\left(\frac{\frac{2}{3}N\bar{E}_k}{V}\right)V = nRT$$

- Rearranging the equation and recognizing that $n = \frac{N}{N_A}$ gives

$$\boxed{\bar{E}_k = \frac{3}{2}\frac{R}{N_A}T}$$

- This equation tells us that the absolute temperate of a gas is directly proportional to the average kinetic energy of the molecules in an ideal gas

- We can also write the equation as

$$\overline{E}_k = \frac{3}{2} k_B T$$

k_B = Boltzmann's constant

$$k_B = \frac{R}{N_A} = \frac{8.31\,\text{JK}^{-1}\text{mol}^{-1}}{6.02 \times 10^{23}\,\text{mol}^{-1}} = 1.38 \times 10^{-23}\,\text{JK}^{-1}$$

Internal Energy

- The internal energy consists only of the random kinetic energy of the molecules
- Therefore if there are *N* molecules of a gas, the total average kinetic energy or internal energy of the gas is

$$U = \frac{3}{2} N k_B T \qquad \text{or} \qquad U = \frac{3}{2} nRT$$