Kinematics

Vectors and Scalars

- Scalar
  - quantities that only contain magnitude and units.
  - Ex: time, mass, length, temperature
- Vector
  - quantities that contain magnitude and direction (as well as units).
  - Ex: displacement, velocity, force, acceleration

Vector Notation

- We represent a vector quantity by drawing an arrow above the letter.

$$\vec{d}$$

Direction

- The direction of the vector can be described in a number of ways:
  - Common terms (left/right, up/down, forward/backward)
  - Compass directions (north, south, east, west)
  - Number line, using positive and negative signs (+/-)
  - Coordinate system using angles of rotation from the horizontal axis

Adding Vectors

- There are two ways that we will use to add vectors:
  - Scale drawings
  - Algebraically

Scale Drawings

- We draw vectors as lines with an arrow head representing the tip of the vector

- The other end of the vector line is called the tail

- Choose an appropriate scale
- Draw a line representing the first vector
- Draw a line representing the second vector starting from the tip of the first vector
- Continue until all vectors are drawn
- Join the tail of the first vector to the tip of the last vector
- Measure the length and angle of the joining line

- Example:
  - Add the following vectors: 5 m/s North and 10 m/s East

Addition Using Algebra

- We could also add these vectors algebraically (really using the Pythagorean theorem)

Position, Distance & Displacement

- Position
  - Where the object is
- Distance
  - The total amount that the object has moved
- Displacement
  - The difference between the final position and the initial position of the object
  - Includes direction

Displacement

- The change in position of an object.
- How far the object is away from its starting position.
- Displacement is a vector quantity.

$$\vec{d} = \vec{d}_f - \vec{d}_i$$

Example 1

- A woman begins at the origin and walks 4 m east and then 3 m west. What is her final position? What is her distance traveled? What is the final displacement of the motion?

Position = 1 m east
Distance = 7 m
Displacement = 4 – 3 = 1 m east

Example 2

- A man begins at a position 2 m east of the origin. He then travels 4 m east and then 3 m west. What is his final position, distance traveled, and displacement?

Position = 3 m east
Distance = 7 m
Displacement = 1 m east

Example 3

- A car begins at the origin, travels 4 km east and then 3 km north. What is the final position, distance traveled, and displacement?

Position = 5 km 37^{o} north of east
Distance = 7 km
Displacement = 5 km 37^{o} north of east

Definitions

- Instantaneous
  - Value at a point, at an instant
    - 2:01
    - Speedometer
- Interval
  - Difference between two points
  - Represented by Δ
    - Time length of class

Velocity

- Speed and direction
- Defined as the displacement divided by time

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$

Acceleration

- How velocity changes with time
- Defined as the change in velocity divided by time

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

A Note about Acceleration

- Acceleration can occur three ways:
  - Speeding up
  - Slowing down (sometimes called decelleration)
  - Changing direction

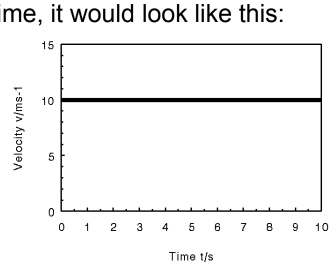
A Note about Signs

- Velocity
  - Sign indicates direction
    - Positive = moving “forward”
    - Negative = moving “backward”

- Acceleration
  - Sign could indicate direction or whether the object is speeding up or slowing down
    - Positive - object speeding up while going forwards (++)
    - Negative - object speeding up while going backwards (+-)
    - Negative - object slowing down while going forwards (-+)
    - Positive - object slowing down while going backwards (--)

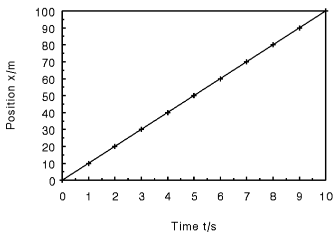
Graphical Representation of Motion

- Consider a car traveling at a constant velocity of 10 m/s. If we were to draw a graph of velocity versus time, it would look like this:



- It is also useful to graph position versus time.
- We will make the decision that when t=0, our position, x, will be 0.
- Since the car is moving with constant velocity, we can easily calculate how far the car will have traveled in 1s, 2s, 3s, etc.

- Plotting this gives us the following graph:

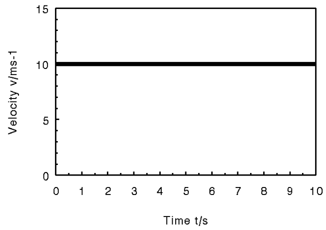


- What is the slope of this line?

$$\frac{\Delta x}{\Delta t} = \frac{90-10}{9-1} = \frac{80}{8} = 10\ m/s$$

- Notice that the slope calculation is exactly the same as our definition of velocity
- We can therefore conclude that the **slope of a position – time graph is velocity**

- Let's go back to our original velocity – time graph

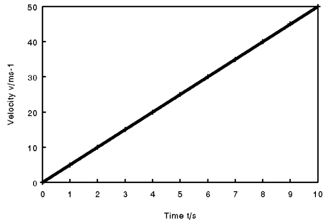


- Calculating the area under the curve gives:

$$v \times t = 10 \times 10 = 100m$$

- Notice that this gives us the total displacement of the car
- Therefore, the **area under a velocity – time graph is displacement**

- Now consider a car that constantly accelerates at a rate of 5 m/s^2 from a velocity of zero
- Graphing the velocity versus time gives us:

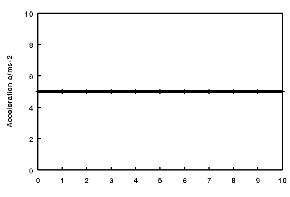


- Calculating the slope of this line gives us:

$$\frac{\Delta v}{\Delta t} = \frac{45-5}{9-1} = \frac{40}{8} = 5\ m/s^2$$

- Notice that this is exactly the same as acceleration
- We can therefore say that the **slope of a velocity – time graph is acceleration**

- Let's graph this car's acceleration versus time



- Calculating the area under the curve gives us:

$$a \times t = 5 \times 10 = 50\ m/s$$

- Notice that the area gives us the final velocity of the car
- This means that the **area of an acceleration – time graph is velocity**

- We can summarize all of this as follows: