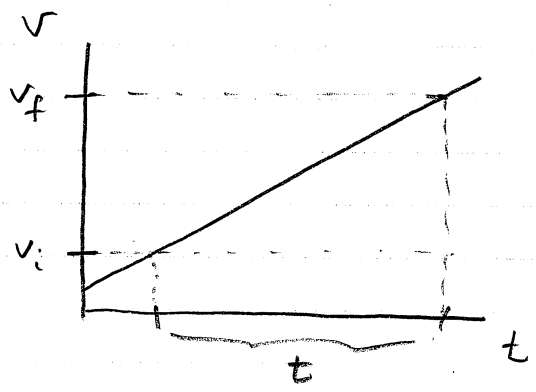
Deriving the kinematic Equations



$$\text{slope} = a = \frac{v_f - v_i}{t} = \frac{v_f - v_i}{t}$$

$$at = v_f - v_i$$

$$\boxed{v_f = v_i + at} \quad (1)$$

$$\text{area} = d = v_i t + \frac{1}{2}(v_f - v_i)\, t$$

but $v_f - v_i = at$

so $$d = v_i t + \frac{1}{2}(at)\, t$$

$$\boxed{d = v_i t + \frac{1}{2} a t^2} \quad (2)$$

$$v_{ave} = \frac{d}{t}$$

$$\left(\frac{v_i + v_f}{2}\right) = \frac{d}{t}$$

$$\boxed{d = \left(\frac{v_i + v_f}{2}\right) t} \quad (3)$$

solve ③ for t and substitute into ①

$$d = \left(\frac{v_i + v_f}{2}\right) t$$

$$t = \frac{2d}{v_i + v_f}$$

$$v_f = v_i + at$$

$$v_f = v_i + a\left(\frac{2d}{v_i + v_f}\right)$$

$$(v_i + v_f)v_f = (v_i + v_f)v_i + 2ad$$

$$v_i v_f + v_f^2 = v_i^2 + v_i v_f + 2ad$$

$$\boxed{v_f^2 = v_i^2 + 2ad} \quad ④$$