

Gravitational Fields

(Why are we attracted to that big bright object in the sky)

Newton's Big Idea

- Newton developed the idea that two bodies exert a force on each other over a distance.
 - Specifically, sun and earth
- He concluded that the force of gravity decreases with the square of the distance, r .

$$F \propto \frac{1}{r^2}$$

- Newton realized that the force of gravity depended not only on distance, but also on the mass of the objects.

$$F \propto \frac{m_{\text{earth}} m_{\text{other body}}}{r^2}$$

- He proposed that if this is true between the Earth and some object, why not between any two objects.

Newton's Law of Universal Gravitation

- Every particle in the universe attracts every other particle with a force that is proportional to their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

$$F = G \frac{m_1 m_2}{r^2}$$

G is the universal gravitational constant, $6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

Example 1

- What is the force acting on a 2000 kg spacecraft when it orbits the Earth at a distance of twice the Earth's radius?
 - Radius of Earth = 6380 km
 - Mass of Earth = 5.98×10^{24} kg

$$F = 4900 \text{ N}$$

Example 2

- A 50 kg person and a 75 kg person are sitting on a bench so that their centers are 50 cm apart. What is the magnitude of the gravitational force each exerts on each other?

$$F = 1 \times 10^{-6} \text{ N}$$

Gravitational Field Strength

- We can use Newton's Law of Universal gravitation to calculate the gravitational field strength (also called gravitational acceleration), g .

$$g = G \frac{m_{earth}}{r^2}$$

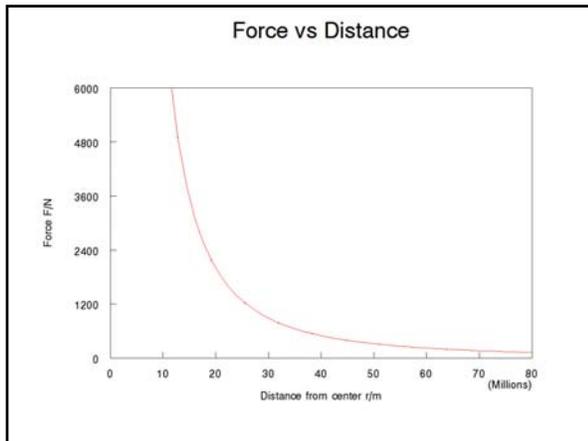
Example

- Mt. Everest is 8848 m above sea level. Determine the value of g at the top of the mountain.
 - Radius of Earth = 6380 km
 - Mass of Earth = 5.98×10^{24} kg

$$g = 9.77 \text{ Nkg}^{-1} \text{ (or ms}^{-2}\text{)}$$

Another Look at Gravitational Potential Energy

- We previously noted that it took work to lift an object off the surface of the earth.
- In more general terms, it takes work to separate two objects.
- Work can be calculated by finding the area under the Force vs Distance curve.
- The work done is equivalent to the Gravitational Potential Energy.



- Force $F = G \frac{m_1 m_2}{r^2}$
- Potential Energy $E_p = -G \frac{m_1 m_2}{r}$
 - Force at an infinite distance is 0
 - Therefore, potential energy at an infinite distance must be 0
 - Potential energy decreases as the distance decreases, therefore E_p must be negative

Escape Velocity

- How fast do you have to go to escape the gravitational field of a planet (or any massive object)?
- The total energy of a moving object, m , near a large stationary mass, M , is

$$E = \frac{1}{2}mv^2 - G \frac{mM}{r}$$

- At a long distance away (∞), then the mass, m , should only have kinetic energy
- Conservation of energy tells us that these two energies should be equal to each other
- Therefore, for the total energy, if
 - $E > 0$: mass escapes and never returns
 - $E < 0$: mass moves out a certain distance but is pulled back
 - $E = 0$: mass just barely escapes
- We use this third case to find the escape velocity

$$\frac{1}{2}mv^2 - G\frac{mM}{r} = 0$$

$$v = \sqrt{\frac{2GM}{r}}$$

- This is the escape velocity for any planet
- Using gravitational field strength, g

$$g = G\frac{M}{r^2}$$

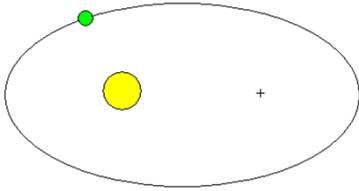
$$v = \sqrt{2gr}$$

Orbital Motion

- Kepler deduced that the planets orbited the sun in elliptical paths from observations made by Tycho Brahe

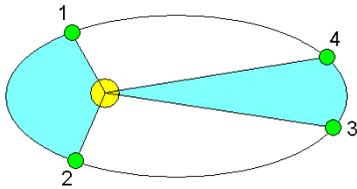
Kepler's 1st Law

- The path of each planet around the Sun is an ellipse with the Sun at one focus.



Kepler's 2nd Law

- Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal periods of time.



Kepler's 3rd Law

- The ratio of the squares of the periods of any two planets revolving around the sun is equal to the ratio of the cubes of their mean distances from the sun.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

Kepler's 3rd Law continued

- Rearranging gives us:

$$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3} = \text{constant}$$

- Newton's law of universal gravitation and his second law of motion provide a theoretical understanding of Kepler's conclusions

- Consider a planet (mass= m) in a circular orbit of radius r , around the sun (mass= M)

$$F = G \frac{mM}{r^2}$$

$$G \frac{mM}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

- This also applies to satellites orbiting Earth (M would be the mass of Earth)

- What if we wanted to use period instead?

$$G \frac{mM}{r^2} = \frac{m4\pi^2 r}{T^2}$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Weightlessness

- Why does an astronaut in a spaceship orbiting the Earth feel weightless?
- There is a gravitational force acting on the spaceship (and thus the astronaut)
- BUT
- The astronaut AND the spaceship are free falling (it just happens to be moving in a circle) and so there are no reaction forces from the floor of the spaceship
