

Work & Energy

Energy

- The ability to do work
- A moving object can do work on another object it strikes
 - hammer on nail
 - cannon ball on wall
- This energy of motion is known as **kinetic energy**

Derivation

- Consider a mass, m , that is moving in a straight line with an initial speed, v_1 . To accelerate it uniformly to speed v_2 , a constant net force F_{net} is exerted on it parallel to its motion over a distance, s .

$$W_{net} = F_{net}s$$
$$F_{net} = ma$$
$$W_{net} = mas$$
$$v_2^2 = v_1^2 + 2as \quad a = \frac{v_2^2 - v_1^2}{2s}$$
$$W_{net} = m \left(\frac{v_2^2 - v_1^2}{2s} \right) s$$

$$W_{net} = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

We define the kinetic energy to be $\frac{1}{2}mv^2$

$$E_k = \frac{1}{2}mv^2$$

So...

$$W_{net} = \Delta E_k$$

This is known as the **Work-Energy principle**

Work-Energy Principle

- The work-energy principle tells us that
 - if positive net work, W , is done on a body, its kinetic energy increases by an amount W
 - if negative net work, W , is done on a body, its kinetic energy decreases by an amount W

Example

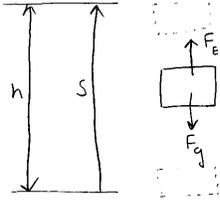
- A 145 g baseball is thrown with a speed of 25 ms^{-1} .
 - What is the kinetic energy of the ball?
 - How much work was done on the ball to make it reach this speed if it started from rest?

Potential Energy

- It is also possible to have potential energy, which is the energy associated with forces that depend on the position or configuration of a body (or bodies) and the surroundings.
 - Wound up clock (work done on spring)
 - Brick held up in the air (gravitational)
 - Propane (chemical)

Gravitational Potential Energy

(most common)



- To lift an object vertically, we must exert a force of at least mg .
- To lift without acceleration a height h , a person must do work equal to the product of the external force and the distance.

$$W_E = F_E s \cos \theta = mgh$$

Gravity also does work

$$W_g = F_g s \cos \theta = -mgh$$

If we allow the brick to freefall from a height, h , its final velocity will be...

$$v^2 = u^2 + 2as$$

$$(u = 0, \quad a = g, \quad s = h)$$

$$v^2 = 2gh$$

It then would have a kinetic energy of...

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m2gh = mgh$$

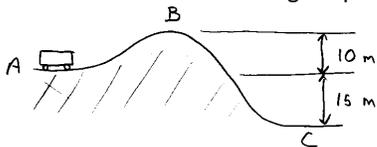
Which is the same amount of work it took to lift it.

Therefore, we define gravitational potential energy as the product of its weight (mg) and its height above some reference point.

$$E_p = mgh$$

Example

- A rollercoaster has the following shape:



- The rollercoaster car has a mass of 1000 kg.
 - Determine ΔE_p from A to B; A to C; B to C with A at $y=0$
 - Determine ΔE_p from A to B; A to C; B to C with C at $y=0$

Force in a Spring

- The force exerted by a spring depends on the amount that it is stretched.
- This is known as Hooke's Law

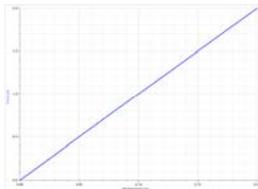
$$F = -kx$$

k = spring constant (different for every spring)
x = the distance that the spring is stretched

Elastic Potential Energy

- A stretched spring can provide energy when it is released
- Therefore it contains potential energy
- We can calculate this amount of elastic potential energy by calculating the amount of work done in stretching the spring.

- We can calculate the work by finding the area under the force-displacement graph for a spring.



$$W = \frac{1}{2}xF$$

But...

$$F = kx$$

So...

$$W_s = \frac{1}{2}kx^2$$

And since work is equivalent to energy...

$$E_s = \frac{1}{2}kx^2$$

Conservation of Energy

- The total energy is neither increased nor decreased in any process.
- Energy can be transformed from one form to another, and transferred from one body to another, but the total amount remains constant.

$$\Sigma E_{before} = \Sigma E_{after}$$

Example

- A stone falls from a height of 3.0 m (initial velocity is 0). Calculate the stone's speed when it has fallen to 1.0 m above the ground.

Energy Transformations

- Types of energy
 - Gravitational potential
 - Elastic potential
 - Chemical
 - Nuclear
 - Heat
 - Light
